

Contents

Preface	ix
Part 1. Invariant Random Matrix Ensembles: Unified Derivation of Eigenvalue Cluster and Correlation Functions	1
Chapter 1. Introduction and Examples	3
1.1. Introduction	3
1.2. Three Examples	3
1.3. Synopsis of the Book	5
1.4. Some General Remarks	6
Chapter 2. Three Classes of Invariant Ensembles	9
2.1. Precise Definitions of the Ensembles	9
2.2. Explicit Form of the Invariant Probability Densities	10
2.3. Separating Out the Eigenvalue Densities: Matrices with Simple Eigenvalues Form Open Dense Sets of Full Measure	14
2.4. Separating Out the Eigenvalue Densities: Computing the Jacobians	18
2.5. Integrating Out Variables Other Than the Eigenvalues	32
Chapter 3. Auxiliary Facts from Functional Analysis, Pfaffians, and Three Integral Identities	37
3.1. Statement of Auxiliary Results	37
3.2. Proof of the Results from Functional Analysis	38
3.3. Proof of the Result on the Pfaffian	42
3.4. Proof of the Three Integral Identities	57
Chapter 4. Eigenvalue Statistics for the Three Types of Ensembles	65
4.1. Computing Expectations: Correlation Kernels	65
4.2. Computing Gap Probabilities	86
4.3. Computing Occupational Probabilities	86
4.4. Computing Correlation and Cluster Functions	89
Part 2. Universality for Orthogonal and Symplectic Ensembles	113
Chapter 5. Widom's Formulae for the $\beta = 1$ and 4 Correlation Kernels	115
5.1. Statement of Widom's Formulae	115
5.2. Proof of Widom's Formulae	119
5.3. Additional Properties of the $\beta = 1, 4$ Correction Terms	130

5.4. General Remarks Concerning the Matrices D_N and ϵ_N and Another Useful Identity among Certain Determinants	135
Chapter 6. Large N Eigenvalue Statistics for the $\beta = 1, 4$ Ensembles with Monomial Potentials: Universality	139
6.1. Introduction and Statement of the Results	139
6.2. Auxiliary Results	146
6.3. Proofs of Theorem 6.7 and Corollary 6.12	154
6.4. Asymptotics of OPs: Matching Formulae	163
6.5. Asymptotics of OPs: Single Integrals and Proof of Theorem 6.55	174
6.6. Asymptotics of OPs: Double Integrals and Proof of Theorem 6.38	178
6.7. Convergence of Derivatives and Integrals of the Christoffel-Darboux Kernel for Monomial Potentials	190
6.8. Differential Equation for the Density $h(x)$ of the Equilibrium Measure and Proof of Theorem 6.51	200
Bibliography	211
Index	217