

Preface

The 2002 Clay School on Geometry and String Theory was held at the Isaac Newton Institute for Mathematical Sciences, Cambridge, U.K., from 25 March through 19 April 2002. It was run jointly by the organizers of two concurrent workshops at the Newton Institute: one on Higher Dimensional Complex Geometry organized by Alessio Corti, Mark Gross and Miles Reid, and the other on M-theory organized by Robbert Dijkgraaf, Michael R. Douglas, Jerome Gauntlett and Chris Hull, in collaboration with Arthur Jaffe, then president of the Clay Mathematics Institute.

This is the second of two books that provide the scientific record of the school. The first book, *Strings and Geometry* [131], edited by Michael R. Douglas, Jerome Gauntlett and Mark Gross, was a proceedings volume and largely focused on the topics of manifolds of special holonomy and supergravity.

The present volume, intended to be a monograph, covers mirror symmetry from the homological and torus fibration points of view. We hope that this volume is a natural sequel to *Mirror Symmetry*, [242], written by Hori, Katz, Klemm, Pandharipande, Thomas, Vafa, Vakil and Zaslow, which was a product of the first Clay School in the spring of 2000. We shall refer to it as MS1. A familiarity with the foundational material of MS1 can be viewed as a prerequisite for reading this volume, and we shall often refer to MS1 for background.

The overall goal of this volume is to explore the physical and mathematical aspects of Dirichlet branes. The narrative is organized around two principal ideas: Kontsevich's Homological Mirror Symmetry conjecture and the Strominger-Yau-Zaslow conjecture. While Kontsevich's conjecture predates the introduction of D-branes into physics, we will explain how the conjecture really is equivalent to the identification of two different categories of D-branes. In particular, we examine how the physics leads us naturally to mathematical concepts such as derived categories and Fukaya categories. We explore the ramifications and the current state of the Strominger-Yau-Zaslow conjecture. We relate these ideas also to a number of active areas of research, such as the McKay correspondence, topological quantum field theory, and stability structures.

As with mirror symmetry in general, these areas have benefited from a remarkably fruitful interaction between mathematicians and physicists. And, over the six year gestation period of the book, a great deal of progress has been made in clarifying and in understanding, and in some cases proving, the original conjectures.

It seems fair to say that to fully appreciate the resulting picture requires having some understanding of both mathematical and physical points of view. Conveying both in the same book has been a challenge and an opportunity. We were not satisfied to simply tell the story twice, once from each point of view. Rather, we attempted a unified presentation, in which both mathematics and physics have their essential insights to provide, explained in a way that physicists and mathematicians can follow without necessarily having all of the foundations of both subjects at their fingertips.

Part of the difficulty in doing this stems from the numerous differences in background and language between physicists and mathematicians; while we feel we have done a great deal to bridge these gaps, it is all the more obvious to us how many gaps remain.

Of course there is a more essential difficulty, which is that the breadth of topics needed to tell the entire story is such that none of the authors are experts in all of them. We have thus divided the main part of the writing while nevertheless striving to unify the book by extensive editing and cross-referencing. The task was carried out by Michael R. Douglas and Mark Gross on the basis of cross-reading and comments made by all of the authors. Michael R. Douglas and Mark Gross take responsibility for the book's success or failure on this level.

Chapter 1 is intended to give a largely physical overview of the topics of the book. Chapter 2, on topological open string theory, is due to Greg Moore and Graeme Segal. An earlier draft of this material appeared as arXiv:hep-th/0609042v1.

Chapters 3 and 5, on the physics of Dirichlet branes, are largely due to Paul Aspinwall, Michael R. Douglas and Anton Kapustin. Parts of this material appeared in arXiv:hep-th/0403166, while §§5.7 and 5.8 are heavily based on Tom Bridgeland's published work on stability structures.

Chapter 4, on representation theory, is largely due to Tom Bridgeland, Alastair Craw, and Balázs Szendrői.

Chapters 6 and 8 are due to Mark Gross, while Chapter 7 is due to Mark Gross and Pelham Wilson.

The entire manuscript was read by Robert Karp and Arthur Greenspoon, both of whom caught numerous imprecisions and unclear points. We also benefited from discussions with and comments by Mohammed Abouzaid, Gary Gibbons, Akira Ishii, Dmitri Orlov, and Bernd Siebert. Several of the authors would also like to thank the hospitality of the IHES, where portions of the book were completed.

Let us again repeat our thanks to those who made the 2002 school possible: H. Keith Moffatt and John Kingman, the directors of the Newton Institute; and its staff, Wendy Abbott, Tracey Andrew, Caroline Fallon, Jackie Gleeson, Louise Grainger, Rebecca Speechley and Christine West.

Finally, let us express our thanks to Jim Carlson and to the staff of the Clay Mathematics Institute, especially Vida Salahi, in helping with the preparation and production of this book. We especially thank Lori Lejeune for providing the artwork in the book.

Michael R. Douglas and Mark Gross
October 2008