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## Directions in Mathematical Quasicrystals

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*Editors*

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## Preface

The experimental discovery of real-world quasicrystalline materials by several groups in the early 1980's brought into question a number of long-standing assumptions in crystallography regarding the relationship between long-range order and periodic order. By now it is clear that there is indeed a very real world of aperiodic long-range order, and there are many fascinating questions to ask and to answer about the vast territory that lies between crystallographic order and random (or amorphous) (dis)order.

Once the dust had settled a bit, after the initial outburst of activities in the world of physics, it turned out that, on the one hand, there were a number of important mathematical predecessors (some of which were painfully reestablished), and that, on the other hand, many new questions emerged that needed (and still need) professional mathematical attention.

The latter started with a NATO ASI workshop at the Fields Institute in fall 1995. Two books with research articles grew out of that: *The mathematics of long-range aperiodic order* (R. V. Moody, ed.), NATO ASI Ser. C: Math. Phys. Sci., vol. 489, 1997, and *Quasicrystals and discrete geometry* (J. Patera, ed.), Fields Inst. Monogr., vol. 10, 1998.

In spring 1998, a meeting on aperiodic order took place in Oberwolfach, and it was the stimulating atmosphere of this meeting together with the large number of new results that led to the decision to edit this volume. It is not meant as a proceedings of this meeting, and in fact it isn't. The reader will find a number of contributions that emerged from talks given at that meeting, as well as a number of additional ones that we have solicited.

All articles were especially written for this volume, with the aim of giving an account of our present knowledge and of the open questions of the field (or at least a substantial part of it). Taken together with the other two volumes, we hope that a rather coherent picture will emerge and that this will serve as a guide and inspiration to those interested in learning more about the mathematics of aperiodic order.

We would like to express our gratitude to our contributors for taking the time and care to produce these accounts of their work and that of their fellow coworkers. Particular thanks also go to Uwe Grimm and Moritz Höffe for providing several figures and for helping us in editing some of the contributions. We also thank our

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