PREFACE

This monograph is based on lectures which were given in two phases. In the fall of 1995 I gave a series of lectures at the Helsinki University of Technology and in October 2001 at the Fields Institute in Toronto.

With this monograph I hope to demonstrate that viewing the resolvent of a matrix as a meromorphic function rather than just analytic outside the eigenvalues gives a lot of new insight. In low rank perturbations the eigenvalues - and pseudospectra - may move dramatically but underneath there is still much which is preserved. Since this has practical implications e.g. to preconditioning, I am trying to present the ideas in a simple and self contained form, accessible for the researchers in the numerical linear algebra community. However, some of the results are more natural to set up in infinite dimensional spaces as the asymptotics is then richer.

The monograph is organized as follows. In the first chapter the resolvent is explicitly written down. The second chapter gives a summary of elementary value distribution theory - without going into the second main theorem. The third chapter then discusses vector valued analytic and meromorphic functions. The main new "tool", the total logarithmic size of a matrix is introduced in chapter four. It is a nonlinear tool for linear algebra and it allows one to generalize the first main theorem from the scalar valued case for matrix valued functions. This is done in chapter five. In chapter six we discuss some applications and show in particular that the growth of the resolvent as a meromorphic function is robust under low rank perturbations. The seventh chapter discusses first operators of the form

$$z \mapsto f(zA)$$

where $f(z)$ is a scalar meromorphic function and A a bounded operator such that its resolvent is a meromorphic function. Another topic discussed is bounds for Krylov methods for solving

$$x = Ax + b.$$

We connect the decay of the bounds for the growth of the resolvent as a meromorphic function and as this is robust in low rank perturbations so are our bounds. Chapter eight gives a new tool into approximation theory. The growth of a meromorphic function is studied by approximating it by rational functions. The results are then applied to Kreiss matrix theorem, power boundedness and other related questions. In the ninth chapter we associate with a given operator valued meromorphic function F scalar functions

$$f_{x,y^*}: \quad z \mapsto y^*(F(z)x),$$

and ask whether there are unit vectors x, y^* such that the growth of F as a meromorphic function can be seen from the growth of f_{x,y^*}. The last chapter gives a link between the defects in value distribution theory and defective eigenvalues of a matrix.

In addition I have included an epilogue and a prologue to explain how I got the ideas in the first place.

I can be reached via e-mail at Olavi.Nevanlinna@hut.fi. Some software is available at URL http://www.math.hut.fi/annex/.

Olavi Nevanlinna
Kirkkonummi, Finland

September 10, 2002