Preface

The aim of this book is to present the Galois theory of homogeneous linear differential equations. This theory goes back to the work of Picard and Vessiot at the end of the 19th century and bears their names. It parallels the Galois theory of algebraic equations. The notions of splitting field, Galois group, and solvability by radicals have their counterparts in the notions of Picard-Vessiot extension, differential Galois group, and solvability by quadratures. The differential Galois group of a homogeneous linear differential equation has a structure of linear algebraic group; hence it is endowed, in particular, with the Zariski topology. The fundamental theorem of Picard-Vessiot theory establishes a bijective correspondence between intermediate differential fields of a Picard-Vessiot extension and closed subgroups of its differential Galois group. Solvability by quadratures is characterized by means of the differential Galois group. Picard-Vessiot theory was clarified and generalized in the work of Kolchin in the mid-20th century. Kolchin used the differential algebra developed by Ritt and also built the foundations of the theory of linear algebraic groups. Kaplansky's book "Introduction to Differential Algebra" made the theory more accessible, although it omits an important point, namely the construction of the Picard-Vessiot extension. The more recent books by Magid and van der Put and Singer assume that the reader is familiar with algebraic varieties and linear algebraic groups, although the latter book compiles the most important topics in an appendix. We point out that not all results on algebraic varieties and algebraic groups needed to develop differential Galois theory appear in the standard books on these topics. For our book we have decided to develop the theory of algebraic varieties and linear algebraic groups in the same way that books on classical Galois theory include some chapters on group, ring, and field theories. Our text includes complete proofs, both of the results on algebraic geometry and algebraic groups which are needed in Picard-Vessiot theory and of the results on Picard-Vessiot theory itself.

We have given several courses on Differential Galois Theory in Barcelona and Kraków. As a result, we published our previous book "Introduction to Differential Galois Theory" [**C-H1**]. Although published by a university publishing house, it has made some impact and has been useful to graduate students as well as to theoretical physicists working on dynamical systems. Our present book is also aimed at graduate students in mathematics or physics and at researchers in these fields looking for an introduction to the subject. We think it is suitable for a graduate course of one or two semesters, depending on students' backgrounds in algebraic geometry and algebraic groups. Interested students can work out the exercises, some of which give an insight into topics beyond the ones treated in this book. The prerequisites for this book are undergraduate courses in commutative algebra and complex analysis.

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