
Contents

Preface	xi
Acknowledgments	xiii
How to use this text	xv
Introduction	xvii
Chapter 1. Arithmetic Functions	1
§1.1. The method of Chebyshev	1
§1.2. Bertrand's Postulate	6
§1.3. Simple estimation techniques	7
§1.4. The Mertens estimates	10
§1.5. Sums over divisors	16
§1.6. The hyperbola method	21
§1.7. Notes	27
Exercises	33
Chapter 2. Topics on Arithmetic Functions	41
§2.1. ★ The neighborhood method	41
§2.2. ★ The normal order method	46
§2.3. ★ The Mertens function	49
§2.4. Notes	55
Exercises	56

Chapter 3. Characters and Euler Products	59
§3.1. The Euler product formula	59
§3.2. Convergence of Dirichlet series	64
§3.3. Harmonics	67
§3.4. Group representations	71
§3.5. Fourier analysis on finite groups	76
§3.6. Primes in arithmetic progressions	83
§3.7. Gauss sums and primitive characters	89
§3.8. ★ The character group	95
§3.9. Notes	99
Exercises	103
Chapter 4. The Circle Method	111
§4.1. Diophantine equations	111
§4.2. The major arcs	116
§4.3. The singular series	123
§4.4. Weyl sums	130
§4.5. An asymptotic estimate	138
§4.6. Notes	144
Exercises	150
Chapter 5. The Method of Contour Integrals	157
§5.1. The Perron formula	157
§5.2. Bounds for Dirichlet L-functions	162
§5.3. Notes	165
Exercises	166
Chapter 6. The Prime Number Theorem	169
§6.1. A zero-free region	169
§6.2. A proof of the PNT	173
§6.3. Notes	177
Exercises	179
Chapter 7. The Siegel-Walfisz Theorem	183
§7.1. Zero-free regions for L-functions	183
§7.2. An idea of Landau	190
§7.3. The theorem of Siegel	193
§7.4. The Borel-Carathéodory lemma	196

§7.5. The PNT for arithmetic progressions	198
§7.6. Notes	205
Exercises	205
Chapter 8. Mainly Analysis	209
§8.1. The Poisson summation formula	209
§8.2. Theta functions	216
§8.3. The gamma function	223
§8.4. The functional equation of $\zeta(s)$	227
§8.5. ★ The functional equation of $L(s, \chi)$	231
§8.6. The Hadamard factorization theorem	235
§8.7. ★ The Phragmén-Lindelöf principle	240
§8.8. Notes	243
Exercises	247
Chapter 9. Euler Products and Number Fields	255
§9.1. The Dedekind zeta function	255
§9.2. The analytic class number formula	262
§9.3. ★ Class numbers of quadratic fields	269
§9.4. ★ A discriminant bound	275
§9.5. ★ The Prime Ideal Theorem	281
§9.6. ★ A proof of the Ikehara theorem	287
§9.7. Induced representations	293
§9.8. Artin L-functions	296
§9.9. Notes	302
Exercises	303
Chapter 10. Explicit Formulas	307
§10.1. The von Mangoldt formula	307
§10.2. The primes and RH	314
§10.3. The Guinand-Weil formula	315
§10.4. Notes	322
Exercises	324
Chapter 11. Supplementary Exercises	327
Exercises	327
Solutions	330

Bibliography	341
List of Notations	357
Index	363