Preface

This monograph is intended as an introduction to some elements of mathematical finance. It begins with the development of the basic ideas of hedging and pricing of European and American derivatives in the discrete (i.e., discrete time and discrete state) setting of binomial tree models. Then a general discrete finite market model is defined and the fundamental theorems of asset pricing are proved in this setting. Tools from probability such as conditional expectation, filtration, (super)martingale, equivalent martingale measure, and martingale representation are all used first in this simple discrete framework. This is intended to provide a bridge to the continuous (time and state) setting which requires the additional concepts of Brownian motion and stochastic calculus. The simplest model in the continuous setting is the Black-Scholes model. For this, pricing and hedging of European and American derivatives are developed. The book concludes with a description of the fundamental theorems of asset pricing for a continuous market model that generalizes the simple Black-Scholes model in several directions.

The modern subject of mathematical finance has undergone considerable development, both in theory and practice, since the seminal work of Black and Scholes appeared a third of a century ago. The material presented here is intended to provide students and researchers with an introduction that will enable them to go on to read more advanced texts and research papers. Examples of topics for such further study include incomplete markets, interest rate models and credit derivatives.

For reading this book, a basic knowledge of probability theory at the level of the book by Chung [**10**] or D. Williams [**38**], plus for the chapters on continuous models, an acquaintance with stochastic calculus at the level of the book by Chung and Williams [**11**] or Karatzas and Shreve [**27**], is desirable. To assist the reader in reviewing this material, a summary of some of the key concepts and results relating to conditional expectation, martingales, discrete and continuous time stochastic processes, Brownian motion and stochastic calculus is provided in the appendices. In particular, the basic theory of continuous time martingales and stochastic calculus for Brownian motion should be briefly reviewed before commencing Chapter 4. Appendices C and D may be used for this purpose.

Most of the results in the main body of the book are proved in detail. Notable exceptions are results from linear programming used in Section 3.5, results used for pricing American contingent claims based on continuous models in Sections 4.7 through 4.9, and several results related to the fundamental theorems of asset pricing for the multi-dimensional Black-Scholes model treated in Chapter 5.

I benefited from reading treatments of various topics in other books on mathematical finance, including those by Pliska [**33**], Lamberton and Lapeyre [**30**], Elliott and Kopp [**15**], Musiela and Rutkowski [**32**], Bingham and Kiesel [**4**], and Karatzas and Shreve [**28**], although the treatment presented here does not parallel any one of them.

This monograph is based on lectures I gave in a graduate course at the University of California, San Diego. The material in Chapters 1–3 was also used in adapted form for part of a junior/senior-level undergraduate course on discrete models in mathematical finance at UCSD. The students in both courses came principally from mathematics and economics. I would like to thank the students in the graduate course for taking notes which formed the starting point for this monograph. Special thanks go to Nick Loehr and Amber Puha for assistance in preparing and reading the notes, and to Judy Gregg and Zelinda Collins for technical typing of some parts of the manuscript. Thanks also go to Steven Bell, Sumit Bhardwaj, Jonathan Goodman and Raphael de Santiago, for providing helpful comments on versions of the notes. Finally, I thank Bill Helton for his continuous encouragement and good humor.

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