Preface

This book is a somewhat expanded version of a graduate course in finite group theory that I often teach at the University of Wisconsin. I offer this course in order to share what I consider to be a beautiful subject with as many people as possible, and also to provide the solid background in pure group theory that my doctoral students need to carry out their thesis work in representation theory.

The focus of group theory research has changed profoundly in recent decades. Starting near the beginning of the 20th century with the work of W. Burnside, the major problem was to find and classify the finite simple groups, and indeed, many of the most significant results in pure group theory and in representation theory were directly, or at least peripherally, related to this goal. The simple-group classification now appears to be complete, and current research has shifted to other aspects of finite group theory including permutation groups, p-groups and especially, representation theory.

It is certainly no less essential in this post-classification period that group-theory researchers, whatever their subspecialty, should have a mastery of the classical techniques and results, and so without attempting to be encyclopedic, I have included much of that material here. But my choice of topics was largely determined by my primary goal in writing this book, which was to convey to readers my feeling for the beauty and elegance of finite group theory.

Given its origin, this book should certainly be suitable as a text for a graduate course like mine. But I have tried to write it so that readers would also be comfortable using it for independent study, and for that reason, I have tried to preserve some of the informal flavor of my classroom. I have tried to keep the proofs as short and clean as possible, but without omitting details, and indeed, in some of the more difficult material, my arguments are simpler than can be found in print elsewhere. Finally, since I firmly believe that one cannot learn mathematics without doing it, I have included a large number of problems, many of which are far from routine.

Some of the material here has rarely, if ever, appeared previously in books. Just in the first few chapters, for example, we offer Zenkov's marvelous theorem about intersections of abelian subgroups, Wielandt's "zipper lemma" in subnormality theory and a proof of Horosevskii's theorem that the order of a group automorphism can never exceed the order of the group. Later chapters include many more advanced topics that are hard or impossible to find elsewhere.

Most of the students who attend my group-theory course are second-year graduate students, with a substantial minority of first-year students, and an occasional well-prepared undergraduate. Almost all of these people had previously been exposed to a standard first-year graduate abstract algebra course covering the basics of groups, rings and fields. I expect that most readers of this book will have a similar background, and so I have decided not to begin at the beginning.

Most of my readers (like my students) will have previously seen basic group theory, so I wanted to avoid repeating that material and to start with something more exciting: Sylow theory. But I recognize that my audience is not homogeneous, and some readers will have gaps in their preparation, so I have included an appendix that contains most of the assumed material in a fairly condensed form. On the other hand, I expect that many in my audience will already know the Sylow theorems, but I am confident that even these well-prepared readers will find material that is new to them within the first few sections.

My semester-long graduate course at Wisconsin covers most of the first seven chapters of this book, starting with the Sylow theorems and culminating with a purely group-theoretic proof of Burnside's famous p^aq^b-theorem. Some of the topics along the way are subnormality theory, the Schur-Zassenhaus theorem, transfer theory, coprime group actions, Frobenius groups, and the normal p-complement theorems of Frobenius and of Thompson. The last three chapters cover material for which I never have time in class. Chapter 8 includes a proof of the simplicity of the groups $PSL(n, q)$, and also some graph-theoretic techniques for studying subdegrees of primitive and nonprimitive permutation groups. Subnormality theory is revisited in Chapter 9, which includes Wielandt's beautiful automorphism tower theorem and the Thompson-Wielandt theorem related to the Sims conjecture. Finally, Chapter 10 presents some advanced topics in transfer theory, including Yoshida's theorem and the so-called "principal ideal theorem".

Finally, I thank my many students and colleagues who have contributed ideas, suggestions and corrections while this book was being written. In particular, I mention that the comments of Yakov Berkovich and Gabriel Navarro were invaluable and very much appreciated.