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# Historical Introduction

Jeremy Gray

Poincaré's creation of a theory of automorphic functions in the early 1880s was one of the most significant mathematical achievements of the nineteenth century. It directly inspired the uniformisation theorem, it led to a class of functions adequate to solve all linear ordinary differential equations, and it focused attention on a large new class of discrete groups. It was the first significant application of non-Euclidean geometry. The implications of these discoveries continue to be important to this day in numerous different areas of mathematics.

In the 1920s several Russian mathematicians were involved in the preparation of an edition of the collected works of N. I. Lobachevskii, one of the celebrated discoverers of non-Euclidean geometry. Probably because Hadamard's long article on the mathematical work of Poincaré had recently come out (Hadamard [1921]), he was invited to contribute to this project. The manuscript he wrote dates from the 1920s. It was translated into Russian by A. V. Vasil'ev, edited by B. A. Fuks, and published in 1951 as volume 6 in the series *The Geometry of Lobachevskii and the Development of Its Ideas*. Vasil'ev was the author of several works, including *Space Time Motion*, a historical account of Einstein's theory of general relativity, and Fuks was a distinguished Russian mathematician. He wrote volume 5 in the same series, and in his book he explains that it is good technical preparation for Hadamard's book. Hadamard's original French text appears now to be lost, and this translation is based on the Russian edition.

The occasion for Hadamard's first visit to the Soviet Union was an official invitation in 1930 to take part in the first All Union Congress of Mathematicians in Kharkov on the occasion of the fiftieth anniversary of the Kharkov Mathematical Society, which Hadamard had joined in 1903. The delegation also contained Montel, Denjoy, Mandelbrojt, and Élie Cartan. Hadamard enjoyed good relations with Soviet mathematicians and knew some personally even before the trip. He had been elected a Corresponding Member of the Russian Academy of Sciences in 1922 and a Foreign Member of the Academy of Sciences (essentially the same body but with a different name) in 1929. He returned to the Soviet Union on another official visit in 1934 and went again after the war, in 1945, which was to cause him difficulties when he wished to attend the International Congress of Mathematicians in Cambridge, Massachusetts, in 1950. For a rich account of Hadamard's life and work, readers may consult the recent book by Maz'ya and Shaposhnikova [1998].

It would be otiose to introduce an account by Hadamard with explanations of the mathematics, even when he has set himself to describe Poincaré's imprecise creation of Fuchsian and Kleinian functions. But a brief historical survey may shed useful light on what Hadamard did and on the choices that he made in setting Poincaré's work before a Russian audience.