

Introduction

Analysis as an independent subject was created in the 17th century during the scientific revolution. Kepler, Galileo, Descartes, Fermat, Huygens, Newton and Leibniz, to mention but a few important names, contributed to its genesis. Questions from mechanics, optics and astronomy played a role in its early days, as well as problems internal to mathematics, such as the calculation of areas, volumes and centres of gravity and the analysis of involved curves. Motion along curved paths under the influence of variable forces became an area of particular interest after Galileo's study of freely falling bodies had led to initial success. Out of this wide variety of efforts there emerged by the end of the 17th century, in the work of Newton and Leibniz, the new mathematical discipline whose history is the subject of the present volume. Very broadly stated, the object of this science is the study of *dependencies among variable quantities*.

Since that time, no other mathematical field has influenced the development of modern scientific thinking as deeply. The basic idea of using differential equations to gain insight into the global behaviour of varying quantities from their (infinitesimal) changes has proved fundamental and fruitful far beyond mathematics and physics and has shaped our overall scientific view of the world, especially our notion of causality. At the end of the 18th century, in fact, most scientists had come to agree that processes in nature (and society) are deterministic and obey laws that may be described in terms of differential equations. Laplace, then the master of mathematical physics, suggested that an omniscient intelligence, enjoying complete knowledge of these laws and of the state of the world at a given point in time, could predict the further development of the world for ever and anon. It was the notion of a *law of nature* that inspired the mathematical concept of *function*, of course, but this notion would never have been as influential if mathematical analysis had not devised such successful methods for the study of functional dependencies.

The development of mathematical analysis has displayed unique vitality and momentum. Newton and Leibniz were thoroughly conscious of the novelty and importance of their creation, yet one can hardly imagine that they could have anticipated how the science they invented would develop in the hundred years following their work. The same might be said for Euler and Cauchy. The depth of change can also be assessed by considering just how far today's scientific thinking has distanced itself from Laplace's determinism.

The present history of analysis seeks to describe this dramatic development in all its dimensions, and to do so satisfactorily a discussion of general trends would be insufficient. Scientific progress is impelled by the solution of concrete problems, and so a reconstruction of general trends in the history of analysis clearly must be complemented by an examination of the specific problems, in all their variety, which both challenged the new discipline and contributed to its growth.

For these reasons it seemed appropriate to produce this book as a collective work of authors who are proven historical experts in specific fields. First drafts of the chapters were initially exchanged among authors and then discussed and coordinated at a conference at the University of Essen. The final versions of the chapters were written after a new round of evaluation and mutual criticism. The resulting volume manages to clarify the conceptual change which analysis underwent in the course of its development, while elucidating the influence of specific applications and describing the relevance of the biographical and philosophical backgrounds.

The book is aimed at a broad audience. Mathematical examples are selected and presented in such a way that they can be understood by any reader with a college background and a certain openness to mathematical argumentation. Those who would like to pursue a topic in more depth will find a comprehensive reference to the sources and the relevant secondary literature. If a reliable English translation of a source is available, it is quoted and used along with the source.

The first ten chapters of the book present a chronology of analysis up to the end of the 19th century. Chapter 1 describes those developments of antiquity which the authors of the 16th and 17th centuries were able to build upon. In describing works on infinitesimal analysis in the period before Newton and Leibniz, Chapter 2 concentrates on the frequently underestimated Cartesian tradition. Chapter 3 directs its attention to the conceptual differences between the approaches of Newton and Leibniz to infinitesimal analysis and the relation of these differences to mechanics. Chapter 4 analyses the change from a geometrical to an algebraic conception of analysis which took place in the 18th century in the works of Euler and Lagrange and which was accompanied by the emergence of the concept of function.

The relation of mathematics to its applications, mainly in physics, is present in every chapter of this narrative history. Two chapters focus directly on this topic; however, Chapter 5 sketches the genesis of analytical mechanics, while Chapter 7 delineates those problems of 19th-century physics, such as potential theory, which led to the fundamental integral theorems of Gauss, Green and Stokes.

The deep conceptual change in analysis brought about in the 19th century by Cauchy and Weierstrass is analysed in Chapter 6. Chapter 8 describes the emergence and flowering of the theory of complex functions; the extensive treatment of this subject is a special feature of the present book. Chapter 9 examines the history of the concept of the integral from Riemann to Lebesgue, the fascinating, almost paradigmatic development of a mathematical concept in which every step was motivated by concrete problems and intentions. Finally, Chapter 10 deals with the foundations of analysis in the second half of the 19th century. Mathematically, it is about the emergence of an adequate theory of real numbers and the genesis of set theory. This development had far-reaching consequences for mathematics and its philosophy and culminated in the so-called foundational crisis.

The chronology just outlined is complemented by survey chapters on subjects which could have been integrated into the narrative only at great sacrifice of clarity. These concern the theory of differential equations (Chapter 11), the variational calculus (Chapter 12) and functional analysis (Chapter 13).

Each chapter contains biographies of one or two mathematicians who were especially influential in the period under discussion.

Some explanation of the references is in order. In principle, we use the style (author year, page), for example, (Cauchy 1825, 50). In the bibliography, however,

the reader will find for (Cauchy 1825) both the original publication and its reprint in Cauchy's *Oeuvres*. In every case the year designates the year of original publication, whereas the page number refers to the last edition mentioned in the bibliographical entry (in the above example, to the *Oeuvres*).

Some references do show two years, for example, (Euler 1755/2000, 53). Here 1755 refers to the year of first publication, whereas 2000 is the year of a translation of this source into English. In such a case, the page number refers to the page in the translation. Two years are also shown when referring to publications of academies, as in (Euler 1753/1755, 234). Here 1753 identifies the annual volume of the academy, while 1755 indicates the year in which it was actually published and the page number refers to the Opera. It will be clear from the context to which of these two cases the double year belongs.

The present *History of Analysis* is a translation into English from the original German book published by Spektrum Akademischer Verlag in 1999. Each author was responsible for the translation (or retranslation) of his chapter. This was no problem for the native speakers of English, of course, while the other authors benefited greatly from the unselfish and generous help of Jeremy Gray, London. Some reviewers of the AMS also made significant but anonymous contributions to the polishing and simplifying of the English in the chapters which were not written by native speakers. Nicole Huelsmann invested a great deal of skill and effort on the technical side, including the production of the indexes. I would like to express my heartfelt gratitude to all those who have been so helpful.

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Hans Niels Jahnke
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