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CHAPTER 1

Introduction

Simon Altmann and Eduardo L. Ortiz

The study of the life and times of Olinde Rodrigues—mathematician, social reformer, banker—presents interesting historiographic challenges. To start with, there is only one published article wholly dedicated to him, and that was in 1925. Also, much of the information about Rodrigues scattered in publications on history, mathematics, and the social sciences is inaccurate. This is, probably, because in each of the activities in which he engaged, Rodrigues was at the margin of the centres of power, so that he did not leave a school of followers or pupils. As a result, what information we have about him is most often found in marginal archives, in some cases deliberately created to reinforce the historical position of his competitors. Past French historians have paid far more attention to Rodrigues as a social reformer than as a mathematician; and mathematicians, because his work was not always attuned to his own time, have taken decades (if not a century) to appreciate its value. As a social reformer, he is presented sometimes as some sort of a failure, because the mantle of the utopian-socialist movement, which he largely recreated after the death of Henri de Saint-Simon, was taken from his shoulders by the charismatic Prosper Enfantin. The historian here has to move with great prudence, since some of the documents that we can use to reconstruct his life have been left by Enfantin and his followers, and a certain amount of bias has to be discounted.

Rodrigues left no papers or archives, and the only published discussion about him probably produced by an eyewitness is from the French historian Clarisse Coignet in 1883. This one article as well as letters from women to the important Paris periodical *Le Globe* (Rodrigues was a precursor of feminism) paint a picture that in some ways contradicts other material. Even for the analysis of Rodrigues as a mathematician, the discreet veil that covers the study of racial problems in some periods of French history presents problems to the historian. Rodrigues was the scion of a Jewish family of long standing in France. We know that, in principle, he could have had access to either the *École Polytechnique* or the *École Normale*, because Altmann, Siminovitch, and Ratcliffe in Chapter 2 show that some Jews were admitted to these schools, at least until 1813; the problem, however, is why Rodrigues, after his brilliant mathematical schooling, did not pursue an academic career. Some of the authorities from whom we asked information claimed that no racial discrimination ever existed in the French educational system, whereas the fact is that after the Bourbon restoration of 1815 no Jewish man (women did not exist for this purpose) tried to follow an academic career, even as a school teacher. Barrie Ratcliffe presents in Chapter 3 conclusive evidence that there were instructions from the highest levels of the educational bureaucracy that Jews would not be acceptable

as teachers: this puts to rest the pious thoughts that it was entirely Rodrigues's own choice to abandon academic mathematics and follow his father's career in banking.

Despite the scarcity of direct documentary evidence on Rodrigues, we present in this book for the first time an authoritative account of Rodrigues's ancestry, of his family, of his family tree, and of his schooling. Chapter 2 summarizes all the information we could obtain on these matters, as well as presenting three hitherto unknown portraits of Rodrigues. A fourth portrait, published only once before in 1925, is also reproduced. Chapter 2 also contains, of course, a brief discussion of Rodrigues's mathematical works, most of which are discussed in more detail in the rest of the book. In order to present a rounded biography of our man, however, Chapter 2 must also deal with Rodrigues's career as a Saint-Simonian social reformer and as a banker. A very useful complete list of all the works by Rodrigues, both mathematical and otherwise, is included in the chapter.

Much of the evidence presented in Chapter 2 comes from derivative sources, often memoirs about contemporaries of Olinde Rodrigues. We have been very fortunate in persuading Barrie Ratcliffe, whose work on the Paris Jewish community in the first half of the nineteenth century is well known, to investigate notarial and other archives in order to obtain as much primary evidence about Rodrigues as possible (again, a problem for the historian, since much archival material was burned during the upheavals in Paris during that period). This Ratcliffe has done, and the result is the splendid Chapter 3, which contains much information about Rodrigues and his life not available in any form until now, providing as well a very useful picture of the Jewish community in Paris at that time. Likewise, Paola Ferruta has given in Chapter 4 another powerful insight into Rodrigues's personality and life through the testimony of his wife Euphrasie, of whom some letters are extant in the archives. Ferruta discusses these in full, as well as two letters from Saint-Simonian colleagues of Rodrigues. These letters illustrate in a dramatic way the stresses and strains to which Olinde was subjected within the Saint-Simonian community that he himself had helped create. These first few chapters provide an insight into the significant effect of discriminatory barriers on the European culture of the period.

The education of Rodrigues, as shown in Chapter 2, was somewhat unorthodox, since after finishing his *Lycée* he moved straightaway onto doctoral work, publishing at the same time six important mathematical papers in, of all places, a journal mainly devoted to the work of the *Polytechniciens*, although he was not registered at that school. Ivor Grattan-Guinness, in Chapter 5, clears up this obscure problem and provides us with a useful background on Rodrigues's education and early research in mathematics.

The next four chapters are devoted to a detailed discussion of Rodrigues's most important mathematical works. One of the few pieces of work for which Rodrigues was fully credited in the mathematical literature was his formula for the generation of the Legendre polynomials which was part of his doctoral thesis. This work is discussed at length in Chapter 6 by Richard Askey. In this article, Askey discusses connections between two apparently unrelated results obtained by Rodrigues. The first one is the well-known Rodrigues formula for Legendre polynomials in terms of weighted derivatives of powers of the binomial (x^2-1), which he had derived in his doctoral thesis. The second result is the generating function for the $n!$ permutations of $\{1, 2, \ldots, n\}$, the coefficients of which are the number of permutations of the

same sequence with k-inversions, as discussed in a paper that Rodrigues published in 1839. Askey, displaying his well-known mathematical virtuosity, introduces in the Rodrigues formula operators more general than differentiation, as well as generalizations of the binomial theorem, thereby establishing new connections between the two results of Rodrigues.

In the late 1830s Rodrigues engaged in some important work on combinatorics, resulting in four papers that are discussed in Chapter 7 by Ulrich Tamm. In 1838 Rodrigues participated in a discussion of some combinatorial problems initiated by Terquem in the *Journal de Liouville*, and Tamm shows that Catalan's contribution to this work has been overestimated with respect to that of Rodrigues. In a paper of 1839, on a subject also first discussed in the *Journal de Liouville* by Terquem, Rodrigues gave the generating function for the enumeration of permutations with a given number of inversions. As often happens with Rodrigues's work, the formula he produced here is frequently wrongly attributed to other mathematicians. In another paper of 1843 Rodrigues gave a very useful approximation for the central binomial coefficient, also discussed by Tamm, who ends his article with an application of the Rodrigues formula for orthogonal polynomials in the enumeration of alternating-sign matrices.

In 1840 Rodrigues produced a remarkable paper on rotations and their composition. This is extraordinary in two ways: it introduced for the first time a parametrization of rotations not by their full angles but rather by their half angles. This was a point that was missed by some of the greatest mathematical minds until the end of the century and whose importance was not properly understood until the advent of quantum mechanics in the 1920s. The second point about which this paper is pioneering is the implicit introduction of two important symmetry groups, the rotation group and the Euclidean group. Although a few references to this paper appeared in the literature, it was not until Jeremy Gray brought it to the attention of mathematical historians in 1980 that its import was fully realized. We are fortunate in having him present an update of his paper in Chapter 8.

The last chapter of the book, by Eduardo L. Ortiz, gives an account of the fortunes during the second half of the nineteenth century of Rodrigues's work on rotations of a sphere with fixed centre. It is now well known that this work is profoundly related to the concept of quaternions, introduced by Hamilton three years after the publication of Olinde's paper. Ortiz follows very carefully the slow development of the acceptance of quaternions in France during that period and produces good evidence that, even when they were incorporated within the mathematical corpus used in France, the work of Rodrigues was largely ignored until 1901.

CHAPTER 2

Olinde Rodrigues and His Times

Simon Altmann, David Siminovitch, and Barrie M. Ratcliffe

The political upheavals in France from the French Revolution until the middle of the nineteenth century led to profound social and intellectual changes, sometimes reversed by later events—and whatever happened in France was propagated throughout continental Europe. The French Revolution opened education to all and emancipated the French Jews; even after the Bourbon restoration of 1815 much of what had been gained still survived. Likewise, the new social structure encouraged new ideas about the organization of communities, about economics and banking, and about the status of workers and of women. No figure of the period is more representative of all these new forces than Olinde Rodrigues (1794–1851): he exerted an active influence in the social and political life of his period and, as one of the first Jewish mathematicians of the century (a title often given to the German Carl Gustav Jacob Jacobi, 1804–1851, although they were both preceded by the Frenchman Olry Terquem, 1782–1862), he was himself an exemplar of the cultural sea-change of his times. Yet, no other major figure of the period has been so much ignored or misrepresented. The seesaw of events in his own life meant that he never had a centre of power that he could call his own, and as a result he was seen by his contemporaries and by later historians through partial and much distorted views, although in his own time he had been recognized as an intellectual leader who exercised considerable influence on his colleagues, mathematicians and social utopians alike. In many biographical entries he is only described as a Saint-Simonian or as a social reformer, totally ignoring the fact that he was a businessman whose economic ideas were influential in the development of the French economy. What is even more remarkable, in the midst of all these activities he produced some mathematical work that was so advanced that its real importance was not recognized until the end of his century—to be again forgotten to the extent that his very name was traduced in the mathematical literature as that of two separate mathematicians, *Olinde* and *Rodrigues*. Even the date of his death was often and inexplicably misrepresented.

No single biographical paper on Rodrigues has been published since (Courteault 1925). Because of this lack of biographical knowledge, we shall try to put together whatever information on his life that we could obtain either from contemporary or from secondary sources, especially historical works on Saint-Simonians and on the Pereire family, whose archives are extant and, since they were close relatives of the Rodrigueses, refer to him from time to time. Given the need to fill in so many gaps in our knowledge of this remarkable man, we shall proceed systematically, first dealing with Olinde's origins and family.

Family

Rodrigues Olinde@Rodrigues, Olinde (1794–1851) Ferdinand and Isabella, as is well known, expelled the Jews from Spain in 1492, and Emmanuel did the same in Portugal in 1496. From the time of the transfer of Bordeaux from English to French suzerainty in 1453, Marrano Jews from the peninsula, persecuted despite their formal conversion to Christianity, began their flight to that town, then the most important French port and an active centre of commerce.[1] After a couple of centuries, many of the Marranos there had openly returned to Judaism and, despite some restrictions placed on the Bordelais Jewish community in 1718, it continued to prosper. It is to Bordeaux that Olinde Rodrigues's great-grandfather, Isaac Rodrigues-Henriques, fled.

Information about Olinde Rodrigues's life and family has until now been not only incomplete but often incorrect. For this reason we now present complete evidence about his family, and we hope to clear up the uncertainties that have led to contradictory or erroneous information in the literature. At the same time, we shall provide a great deal of new genealogical material. Olinde Rodrigues's great-grandfather, Isaac Rodrigues-Henriques, was born in Spain, and emigrated to Bordeaux probably in the first quarter of the eighteenth century. Fortunately, excellent work has been done on the Bordelais Jews, and we have been able to reconstruct most of the Rodrigueses' family tree. (See Figures 1 and 2. In order to simplify the presentation of these figures, we do not include children who died in their infancy.)[2] Further details about Olinde Rodrigues's siblings, as well as about his children, until now unknown, are shown in Figure 2.[3]

In the study of the Rodrigues-Henriques family, difficulties often arise because of the frequent repetitions of given names, which can create confusion. Even more, there is another large and distinguished family by the same surname which had long been thought to be entirely unrelated, although some cross-links between both families exist.[4] It is now believed that both Rodrigues-Henriques families have a common root, Moÿse Rodrigues-Henriques, who married Rachel Mendès Campos, and whose children include both Isaac 'La Poudrayre' and Abraham Rodrigues-Henriques (c. 1670–c. 1767). It is only recently that notarial documents have come to light that show Abraham and Isaac, who were believed to be unrelated (probably because of a family feud that kept them apart), to be brothers. Abraham married Rachel Fernandes, and their issue originated the large Rodrigues-Henriques family now thought to be one of the two branches of the family with this name.[5]

[1] The history of the French Jewish community is very well given in (Benbassa 1999) and (Nahon 1989, pp. 46–74).

[2] Information for the larger part of Figure 1 comes from (Cavignac 1987, pp. 111–112, 191, and 167), although the names of Olinde's six sisters are from (Ratcliffe 1972, p. 192) and those of Olinde's aunts have been kindly provided to us by Dr. Patrice Assouad. We differ from Cavignac in the date of death of Olinde Rodrigues, which he gives as 26 December 1850, an erroneous date albeit often quoted in the literature. We quote in the family tree, instead, the date 17 December 1851, as registered in the archives of the cemetery of *Père-Lachaise*, Paris, where Olinde is buried.

[3] Constructed from the archival sources listed below as well as the files kindly provided to us by Dr. Patrice Assouad (a genealogist in Paris studying the Rodrigues family, whose wife is a direct descendant of Olinde Rodrigues).

[4] Gustave d'Eichthalfor instance, a close friend of Olinde Rodrigues, married Felicité Cécile Rodrigues-Henriques, of the Abraham Rodrigues-Henriques family.

[5] We are grateful to M. Paul M. Siméon for this information. More details may now be found from the website of the Rodrigues-Henriques family: www.nebuleuse-rh.org.

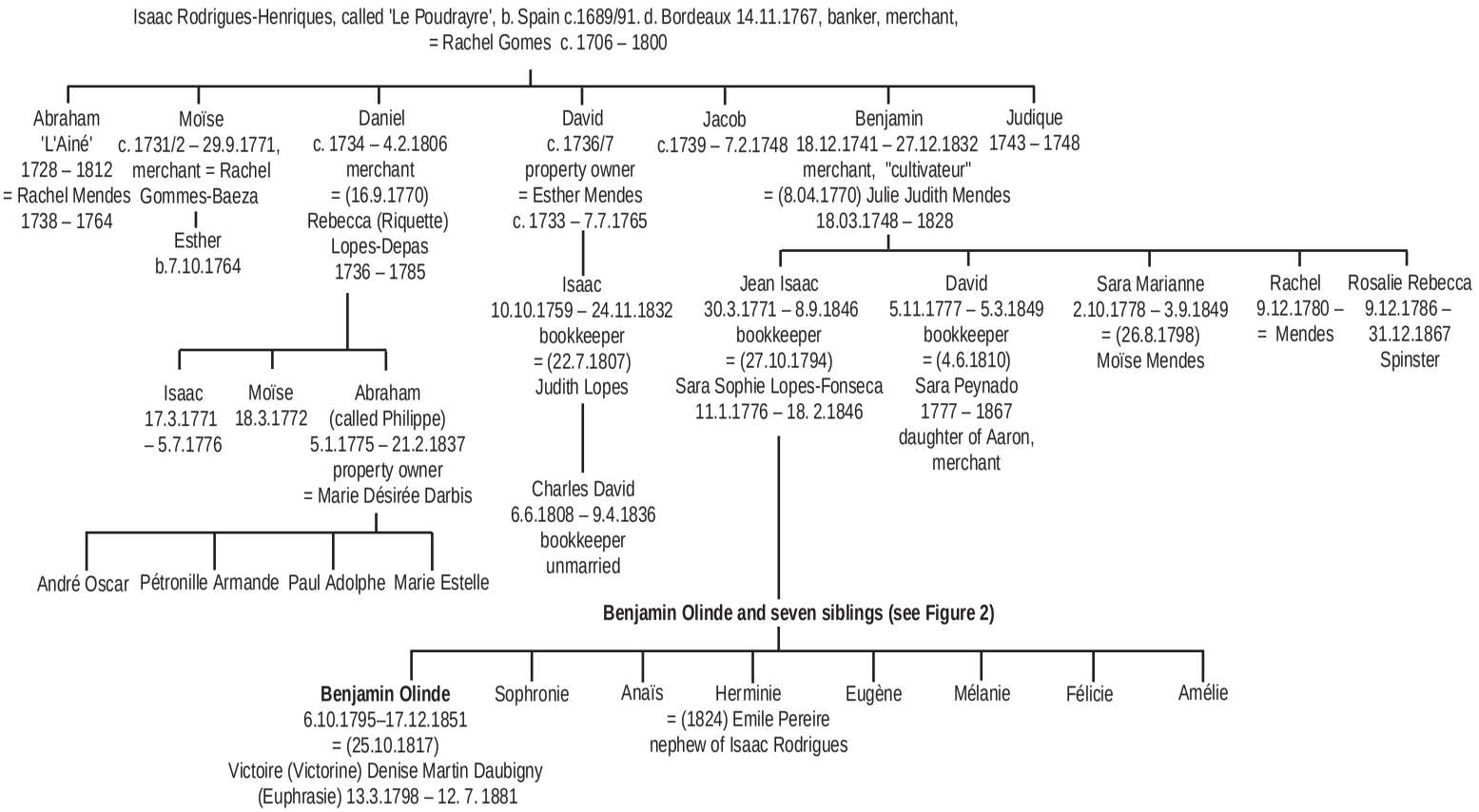

Figure 1. The family tree of the Rodrigues family until Olinde's generation.

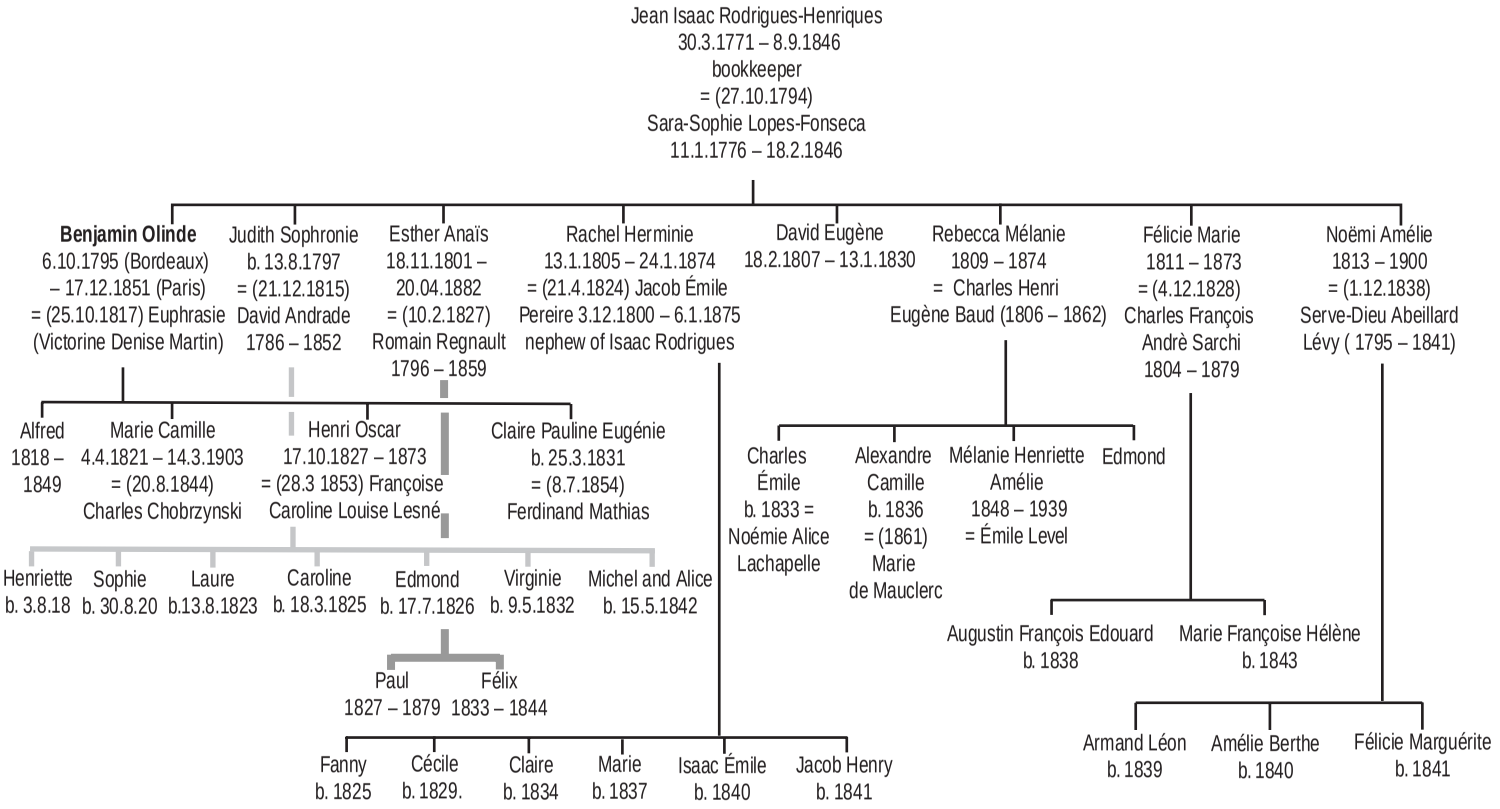

Figure 2. Olinde Rodrigues's family and its descendants.

At about the time of Olinde Rodrigues's birth on 6 October 1795 the Sephardic community in Bordeaux (that is, that composed of Jews of Spanish or Portuguese origin) contained three notable families, the Gradis, the Raba, and the Rodrigues families, and, as with all such communities, was closely knit through intermarriage. (For the Gradis family, see footnote 51.) Another important family, the Pereires, would later be closely connected to the Rodrigueses in Paris: Émile Pereire (Jacob Émile Pereire, 1800–1875) was both a cousin (his mother Rebecca, married to Isaac Rodrigues-Pereire, was a sister of Olinde's mother) and a brother-in-law of Olinde, as can be seen in the family tree. A word or two has to be said about surnames. Rodrigues is a name common in Portugal, whereas in Spain the spelling Rodriguez is the favoured one. Probably because of this, the Spanish *Enciclopedia Ilustrada Espasa-Calpe* claims a Portuguese origin for the family, but we have found no evidence for this. In discussing such an origin for the Rodrigueses, it must be appreciated that at that time in France '*Portugais*' was used as a synonym for Sephardic.[6] In any case, the French were given to modifying name spellings, *Rodriguès* being a form sometimes used for Olinde's family. Also, the Pereires, for example, must have been originally Pereira, and transformed into Pereire in France, where this spelling is still used, although Péreire is also common for this family. Finally, the 'Henriques' of Rodrigues's great-grandfather's name was soon lost, and Olinde himself never used his given name, Benjamin. In fact, the name 'Olinde' was assumed by him at a later stage, since it does not appear on his birth certificate.[7] Regulations were established in 1807 requiring Jews to modify their family names to avoid confusion, owing to many people sharing the same patronymic (Halévy 1863, p. 6). Soon after, it was required of Jews for similar reasons to add to their given names others of French origin. The Napoleonic decree of 20 March 1808, usually called the Decree of Bayonne, required all French Jews to adopt fixed and definitive first and family names and to register them between September and December 1808. We know for certain that after this time Benjamin Rodrigues became Olinde Rodrigues or Benjamin-Olinde Rodrigues: in the *Registre de Consistoire de Paris* of 1809/1810 (now in New York) our man is registered under the single first name of *Olinde*.[8] In any case, assumed names were not unusual at that time: his wife, Euphrasie, (who was not Jewish and thus was not affected by the decree of Bayonne) was in fact called Victorine Denise Martin on their marriage certificate in 1817, in which Rodrigues himself is named as Benjamin-Olinde.[9] Olinde, of course, is not a common name, but it is clear that Isaac Rodrigues was a well-read man who decided to attach to his children non-Jewish second names that would not coincide with those of the Saint's Calendar. Thus *Olinde*, *Sophronie*, and *Herminie* (the latter two being the names of two of Olinde's sisters) are characters in *Gerusalemme Liberata* by Torquato Tasso (1544–1595), the episode referring to Olinde and Sophronie having been translated (as always with Gallicized Italian names) by Jean-Jacques Rousseau in 1781. Likewise, the assumed names of Olinde's other sisters come from literary or musical sources.[10] It is not unlikely that, faced with the need to find

[6] See (Cavignac 1991, p. 279). Relations of the Rodrigues family, the Pereires, hail however from Portugal: Jean-Abraham Rodrigues Pereira (1674–1735) was born in Chacim.

[7] Olinde's birth certificate is given in full in (Courteault 1925, p. 152).

[8] We owe this information to Mme. Stéphane Toublanc.

[9] See Chapter 4 by Paola Ferruta in this volume.

[10] We owe this information to the vast set of genealogical notes about the Rodrigues and Pereire families generously made available to us by Dr. Patrice Assouad. The names *Olinde,*

non-Jewish names for the registration of 1809, Isaac Rodrigues made a wholesale use of such literary and artistic sources as he had at hand.

The profession of bookkeeper (*teneur de livres*) which following Cavignac is quoted for Jean Isaac Rodrigues-Henriques, Olinde's father, in the family tree might not reflect the true status of this interesting man (Ratcliffe 1972, p. 193). On Olinde's birth certificate (Courteault 1925) he is described as merchant (*négociant*). It must be appreciated that at that time and in some financial establishments, bookkeepers were not mere accountants but, acting also as cashiers, had considerable financial clout in the firms for which they worked. In any case, although Isaac Rodrigues worked in Bordeaux for fifteen years as a bookkeeper for the firm of André Aquart, when he moved to Paris later in the 1790s, he worked as exchange agent for the banker Fould (Autin 1984, p. 22) and published a treatise and a manual on accountancy in 1804 and 1810, which include work on banking. Later, he became an independent stockbroker (*kerb-broker*), although outside the Paris Bourse. Isaac Rodrigues had a major role in the Sephardic community of Bordeaux and indeed of France as a whole. After the Revolution, the Bordeaux Jews petitioned the States-General in Paris for emancipation and citizenship, which was granted in 1790 and extended to all French Jews in 1791. Napoleon, who felt that his prejudices against the Jews required revision, summoned an assembly of Jewish notables in 1806, in which Isaac Rodrigues participated as lay member for the Seine department.[11] This *Grand Sanhedrin*, as it was called, proved to Napoleon's satisfaction that Judaism and loyalty to the state were not incompatible, a result for which Isaac's contributions were significant. Although not really well-off, he was certainly not poor: the dowries of his six daughters amounted to a total of 37,575 francs, but Olinde received only 1,500 francs on his marriage (Ratcliffe 1972, p. 193). To compare, during the First Empire a house in Bordeaux could be bought for around 35,000 francs. The Rabas, the richest family of Bordeaux, on the other hand, were worth some 2,000,000 francs in 1827. (See Cavignac 1991, pp. 185 and 253.)

It must be stressed that Isaac Rodrigues must have had a significant influence in Olinde's life in providing him with close contacts with the Parisian Jewish community, both in financial circles, such as his relatives the Pereires, and intellectual circles, such as the Halévy family, who counted composers and writers amongst its members. He moved in fact, probably around 1830, to an apartment on rue Montholon (present 9*ème arrondissement*) directly below the Halévy's apartment on the third floor, often visited by the brothers Émile and Isaac Pereire (Halévy 1863, pp. 16–17). Here Olinde also frequented Jacques-François-Fromental-Elias Halévy, best known as the composer of the opera *La Juive*, and who in 1842 married Léonie Rodrigues-Henriques, Olinde's cousin.[12] Olinde had six sisters, one of whom, Rachel Herminie, married a first cousin, Jacob Émile Pereire. Olinde's only

Sophronie, Herminie used by Tasso are stated by editors of his work to be probably 'nomi immaginari' (personal communication from Professor John Woodhouse). Such choice of names shows the degree of acculturation of Olinde's parents. Tasso was popular in France at the end of the eighteenth century through such works as the five-act play *Olinde et Sophronie*, written by Louis-Sébastien Mercier in 1771. The name *Anaïs* for Olinde's sister born in 1801 is that of the principal character in the opera *Anachréon chez Polycrate* by André Modeste Grétry, first performed in Paris in January 1797, only months after the arrival of the family in Paris.

[11] See (Nahon 1989, p. 65) and (Ratcliffe 1972, p. 197).

[12] See (Locke 1986, p. 95). Eventually, Léonie and Fromental Halévy's daughter married Berlioz.

brother Eugène, to whom he was very close, was delicate, suffering from asthma, and died just before he reached the age of 23 (Courteault 1925, p. 156.)

Despite the central role of the Rodrigueses in the Jewish community, they were no longer observant, reflecting a very common attitude within the recently emancipated Jewish élite. Olinde's wife, Euphrasie, was in fact a Roman Catholic.[13] They were very young when they married in 1817 (Olinde, 22, and Euphrasie, 19), and they lived at 26 rue de l'Echiquier.[14] They had four children, two males, Alfred and Oscar, of whom we only know that the second married Françoise Lesné in 1853 and that he was a stockbroker in the Bourse from 1861 to 1873 (Ratcliffe 1972, p. 193.) Olinde's elder daughter, Marie Camille, outlived her father by more than fifty years, and he also had a younger daughter, Claire Pauline Eugénie. Details of their marriages may be seen in the family trees given in this chapter.

Education

Isaac Rodrigues was one of the first Jews to avail himself of the new freedom of sending his sons to the *Lycées* in Paris, where both Olinde and later Eugène became pupils. Olinde enrolled in 1808 in the *Lycée Impérial* (now the *Lycée Louis-Le-Grand* on the Boulevard Saint Michel). We know that at 17, in 1812, the precocious Olinde was already a class assistant (*maître d'études*) in the mathematics course run in the *Lycée Napoléon* by C. Dinet,[15] who had some years before been the teacher of Augustin-Louis Cauchy (1789–1857), later the leading mathematician of his time. Michel Chasles, who became a famous geometer, was one of Olinde's contemporaries at the *Lycée Impérial*, and got the second accessit at the competition at the end of 1811, the first going to Rodrigues, who had already competed the year before (Bertrand 1902, pp. 39–40.) This competition served also for entrance to the *École Polytechnique*, which had been founded in 1794 very much under the influence of Gaspard Monge, great geometer and close collaborator of Napoleon, and where Rodrigues's teacher, Dinet, had been a member of its first cohort.

The school was a high-calibre military establishment, with luminaries such as Arago and Cauchy amongst its first pupils, and Dinet was himself an examiner for admissions to it. Here we have a problem. Bertrand (Bertrand 1902, pp. 39–40) cryptically writes that Olinde 'could not compete except for a prize,' a phrase that could easily be interpreted as signifying that his Judaism prevented him from entering the school. In 1798 Abraham Gabriel Mossé, however, was admitted to the *École Polytechnique*,[16] the first Jew so accepted, and even in 1813 a Jewish friend of Rodrigues, Myrtil Maas, gained admission to it,[17] although he preferred the *École Normale*, which had also accepted him (Ratcliffe 1972, p. 195). So, contrary to claims often made, the *École Polytechnique* was not totally closed to Jews at that time, and it is not clear why Olinde Rodrigues does not appear to

[13] See (Ratcliffe 1972, p. 197) and (Ratcliffe 1971, p. 1231).

[14] Later they lived on 123 Boulevard Pereire and in 1844 on rue Neuve des Mathurins (personal communication from the files of Dr. Jacques Béjot, Paris). At the time of Olinde's death they were at rue d'Amsterdam.

[15] See (d'Allemagne 1930, p. 29). The *Lycée Napoléon* is now the *Lycée Henri IV* at the *place de la Contrescarpe*.

[16] See the students' register in (Marielle 1855, p. 164).

[17] See (Marielle 1855, p. 146). Maas's name is here spelled *Mirtil*, as it also is in the *École Normale* register (Dupuy and Perrot 1895), but the spelling *Myrtil* is frequent in the literature.

have been admitted to it.[18] Pinet (Pinet 1894), in a comprehensive article on the Saint-Simonian *polytechniciens*, mentions Rodrigues only once and certainly not amongst the alumni. It is often stated that he was a pupil at the *École Normale*, but this is false: the exhaustive centennial memoir edited by Dupuy and Perrot gives Maas as a pupil on p. 671 (Vol. 1), but Rodrigues's name never appears in it.[19]

Why, unlike Maas, did Olinde not apply for entrance at the *Polytechnique*? He was already the teaching assistant in Dinet's class at the *Lycée Napoléon*, and it is just possible that Dinet advised him that he was ready to concentrate on mathematical research, directing him for that purpose to the new *Université de Paris* founded in 1808. There is no doubt, however, that he held close connections with the *École Polytechnique*, perhaps through Dinet's influence. In 1813 he taught mathematics to Prosper Enfantin (Booth 1871), who was a pupil there and later played an important role in Olinde's life, but this was probably a private arrangement. It is frequently said that he was a *répétiteur* (tutor) in mathematics at the school, but there appears to be no foundation for this assertion: both Marielle (Marielle 1855) and Hachette (1813) have full lists of *répétiteurs* on which Rodrigues's name never appears. In 1813 Rodrigues obtained his *licence ès-sciences* and the corresponding dissertation was published by the *École Polytechnique*,[20] but such a title could not have been granted by that institution, its degrees being only on technical or military subjects (Pinet 1894). We must therefore assume that immediately after graduating from the *Lycée Napoléon* in 1812, Rodrigues found a home at the *Université de Paris* and that his *licence ès-sciences* had been obtained under the aegis of the *Faculté des Sciences* of Paris, as was certainly the case for his doctorate, also published in the same way.

For about two or three years starting from 1813, Rodrigues was engaged in mathematical research. This must have been so, since in that period he not only obtained his *licence* but from 1814 to 1816 he published six mathematical papers (see the bibliography below). At the same time, from 1814, he was still a mathematics tutor (*maître d'études*) in Dinet's class at the *Lycée Napoléon*.[21] His research was presented to the University of Paris, where he was granted the degree of *docteur ès-sciences* in 1815.[22] The *Faculté des Sciences* required two theses to be presented for the doctorate,[23] although they were not expected to be of a very high standard. Rodrigues's theses were amongst the first presented to the university,

[18]The comprehensive lists given by Marielle (Marielle 1855) never mention Rodrigues's name in any capacity. The head of the school's archives, Mme. Claudine Billoux, has kindly confirmed to us that there is no reference whatsoever in them to Rodrigues, although he might have attended classes there purely as an unregistered '*auditeur*'.

[19]See (Dupuy and Perrot 1895). Also, the secretary of the *Association des anciens élèves de l'École Normale Supérieure* has kindly confirmed to us that Rodrigues's name is not given in the list of *normaliens*.

[20]See (Rodrigues 1813), republished in (Hachette 1808, pp. 36–37).

[21]Unpublished notes by Charles Lambert at the *Bibliothèque de l'Arsenal*.

[22]See (Rodrigues 1815*a*). The entry for this publication in our bibliography is quoted from (Ratcliffe 1972, p. 195), where Rodrigues's degree is given as *docteur ès-lettres*, whereas in all his later publications in (Hachette 1808) he is listed as *docteur ès-sciences*.

[23]See (Grattan-Guinness 1990, p. 109). We are grateful to Professor Grattan-Guinness for drawing our attention to this point.

and his first one was published in (Rodrigues 1816*f*), whereas the second part is probably (Rodrigues 1816*a*).[24]

It is not clear who were his mathematical mentors during that period, but the name of Sylvestre-François Lacroix, a former pupil of Gaspard Monge at the *Polytechnique*, is the most likely one. Admittedly, Lacroix was appointed professor of mathematics at the Sorbonne only in 1815, but he is mentioned in a footnote of Rodrigues's thesis both as dean of the faculty and as chairman of the examining committee. This thesis contains the only piece of mathematics for which Rodrigues was ever properly credited, the famous 'Rodrigues formula' for the Legendre polynomials, used and named in this fashion to this day, although until Hermite discovered this work, it was called the Ivory and Jacobi formula.

By the time Rodrigues got his doctorate, the good years for the reformers were coming to an end and with them any possibility of Rodrigues becoming an academic mathematician. After the 1815 Restoration the Catholic hierarchy took control of educational and academic institutions: Gaspard Monge was removed from his post at the *École Polytechnique* that he had helped create and from the Academy as well, and he was replaced by Cauchy, undoubtedly a greater mathematician but with strong Jesuit connections. Whatever Jewish mathematicians there were could not obtain teaching positions, and most of them abandoned the subject or used their mathematical abilities in applied work for private enterprises.[25]

Under these circumstances, it is amazing that Rodrigues retained his interest in mathematics for most of his life. In the first few years around the time of his doctorate he worked in collaboration with his friend Myrtil Maas, who had done very well as a mathematician in the *École Normale* but after graduation had to accept the fact that academic jobs were in practice closed to Jews. In 1818 Maas returned to Paris, where he had employment in the recently founded *Compagnie Générale des Assurances sur la Vie et Contre l'Incendie*. He early formed a committee, of which Rodrigues was a member, to control the actuarial side of the company, and the principles that they established were soon adopted by all the French life insurance companies. Maas and Rodrigues created the statistical tables for the *Union* company of which Maas was the first director (Ratcliffe 1972, p. 195). In 1818–1820 he and Maas also used their mathematical training to draw up the actuarial tables that would be used by all French insurance companies through the century, tables that would eventually be published in 1860 and—another sign of their continuing value—that would go through no fewer than seven editions up until 1933 (Pereire 1860).

Despite the frustration of any ambitions he might have had to become an academic mathematician, Rodrigues did not allow his considerable ability to go unused. We shall first sketch his mathematical work, a backbone to his life.

Olinde Rodrigues: The mathematician

It should be quite clear by now why in modern times there were no Jewish mathematicians in Europe until the beginning of the nineteenth century. Although Rodrigues was first, Jacobi (1804–1851) in Germany was more fortunate in being able to become a professional mathematician, and he thus occupies a more central

[24]See the article by Grattan-Guinness in Chapter 5.

[25]For evidence of the official impediments for Jews to enter the academic world in this period, see Chapter 3 by Barrie Ratcliffe.

place in the mathematics of the first half of the century. This was a crucial time in the history of mathematics, Gauss, Euler, D'Alembert, Cauchy, and other luminaries having prepared the ground for the creation of modern mathematics.[26] The importance that the subject acquired is shown by the founding of the first two purely mathematical journals in the world. An old pupil of the *Polytechnique*, Joseph-Diaz Gergonne, founded in 1810 the *Annales de Mathématiques Pures et Appliquées*, but this was soon discontinued and was revived as the *Journal de Mathématiques Pures et Appliquées* by Joseph Liouville in 1836, an important publication most often cited as *Journal de Liouville*. August Leopold Crelle, in Germany, founded in 1826 the *Journal für die reine und angewandte Mathematik*, also normally cited as the *Crelle Journal*.

Rodrigues's contributions were somewhat scattered across time because of the nature of his extensive activities outside mathematics and can roughly be grouped into three clusters,[27] the first of which consists of nine publications. The first two works listed in our bibliography are his licence and doctoral theses, respectively, later published as we shall describe, and the third, on double integrals and radii of curvature, is related to the thesis for Rodrigues's licence. The next six works were all published in the *Correspondance sur l'École Impériale Polytechnique*,[28] although this publication was mainly devoted to the work of pupils and teachers of the school, with which Rodrigues, as we have seen, did not have any formal affiliation. Nevertheless, 'remarkable' pieces of work from external authors could also be published there (Billoux 2001). The first of these six works, (Rodrigues 1816*a*), is interesting mainly as a forerunner of his major 1840 paper, and the second, (Rodrigues 1816*b*), was his dissertation for the *license ès-sciences*, (Rodrigues 1813), in which he studied lines of curvature, extending the work of Monge and of his student Charles Dupin. Rodrigues was able to obtain a formula to relate the length of a radius of curvature and the coordinates of the centre of curvature which Gaston Darboux, the great French differential geometer, remarked some sixty years later was fundamental in the theory of lines of curvature.

The next two memoirs (Rodrigues 1816*c*, 1816*d*) are concerned with the equations of motion for a particle, a central problem in mathematical physics, for the treatment of which Lagrange had formulated a fundamental principle, called the principle of least action. Its application, however, was obscure, and the second of these papers showed for the first time how to obtain Lagrange's equations of motion for a particle by very careful application of the calculus of variations, then a fairly new method. Rodrigues's role as the invisible mathematician of the century, alas, prevailed, and this work was ignored. (Remember that he never had any pupils whose careers would have been enhanced by quoting the works of the master.)

[26]The mathematical developments in France in this period are very well-reviewed by (Grattan-Guinness 1981, 1990).

[27]See (Royal Society 1871) for a list.

[28]See (Hachette 1808). The contributions that appear here were periodically issued in separate numbers and were later bound; we list these papers by the dates of the corresponding bound volumes, but we also give the dates of the corresponding numbers. (It should be noticed that the third volume in the *Bibliothèque Nationale* is incomplete and contains Number 1 only, the date therefore being given as that of that part, 1814.) The dates of the numbers that we give in the bibliography agree with those used by Professor Grattan-Guinness in Chapter 5, which will help the reader to compare our list with his. We are very grateful to Professor Grattan-Guinness for discussion on this point.

Even as soon as 1837 Siméon-Denis Poisson (1781–1840) (the Poisson of the famous statistical distribution), remarked that '[t]oday, this principle of least action is nothing but a useless rule,' which shows not only his ignorance of Rodrigues's work but also how hard it was for people to understand what Olinde had intuited, that is, the central role of this principle in physics—as later testified by the work of Jacobi and of the famous British mathematical physicist Sir William Rowan Hamilton (1805–1865). No nineteenth-century mathematician, with the later exception of Darboux and of Joseph Bertrand, appears even to have noticed this important work, although soon after the turn of the century Jourdain (Jourdain 1908) thought it sufficiently important, as the title of his book shows, to group it with the work of Lagrange and Jacobi. Years of slumber followed until this mathematical gem was rediscovered independently by the Canadian-born Cambridge mathematician Edward Routh in the 1870s.

These two memoirs are followed by the paper (Rodrigues 1816*e*), in which Olinde consolidates the work of his dissertation on lines of curvature (Rodrigues 1815*b*). They are important papers which follow on and considerably extend the work of Monge and Binet. Anticipating Gauss, he used a spherical mapping of a surface, studied the ratio of the areas of the corresponding surfaces, and arrived at a quantity later called the total Gaussian curvature, which he showed equals the product of the principal curvatures. As another example of the credit Rodrigues deserved going to others, this measure of curvature was credited to Gauss, although it appears not to have been known by the latter. (See (Laptev and Rozenfel'd 1996, pp. 6 and 9).) His doctoral thesis, (Rodrigues 1815*a*), was published in (Rodrigues 1816*f*), and it is the only one of his works for which he did eventually obtain proper credit. In studying the attraction of spherical bodies, one has to use so-called spherical harmonics, relatively new when Rodrigues worked on his thesis (they had been invented by Adrien Legendre in 1784). The core, so to speak, of these functions are certain polynomials, called the *Legendre polynomials*. They were fairly well known in Rodrigues's time, but there were no closed formulae to derive them, and he invented an ingenious way to obtain one such formula, called the *Rodrigues formula*, his only lucky discovery in the sense that his name remained attached to it, not without some trouble, as we have mentioned. (See also Chapter 6 by Richard Askey.) The procedure that Rodrigues devised, based on what he called *generating functions* (which had already been used in statistical problems), was later found most useful in deriving similar formulae for many polynomials of interest in mathematical physics.

We must now jump a quarter of a century to reach Rodrigues's next cluster of five mathematical contributions (1838–1840), a quarter of a century that was nevertheless immensely productive in other ways: amongst other things, he became a man of business, with various degrees of success. Given this large gap of time, it is virtually impossible to guess Rodrigues's motivation for involving himself in this research, although it is quite clear that around 1838 he became interested in combinatorial problems, (Rodrigues 1838*a*, 1838*b*, 1838*c*, and 1839) all being concerned with this subject. By far, the most important of these four papers is the last one. Clearly, Rodrigues was keeping abreast of mathematical research by reading the journals of both Liouville and Crelle, in which two papers appeared in 1838 on a combinatorial problem involving the inversion of permutations. Rodrigues solved the problems treated in these papers by again defining some generating

functions, which had been so important in his work on the Legendre polynomials, and it is possible that he was inspired to do this work when he recognized the possibility of their use in this case also. Again, this paper was so assiduously ignored for more than a hundred years that repeated attempts appeared in the literature to solve the problem about which Rodrigues had given the final word. It was not until well into the second half of the twentieth century that this paper was rediscovered by Carlitz (Carlitz 1970).

The next paper that we must now consider is one of the most remarkable pieces of work done by Rodrigues, in which he goes back to the subject of his very first work, rotations, but now of a sphere with fixed centre. The year is 1840, and all we know about him from the historians of the period, who knew nothing about Rodrigues as a mathematician, is that in that year he was 'speculating at the Bourse' (Booth 1871, p. 216), although he was also much concerned about the legislation for the *Banque de France*, as we shall see. The story starts with one of the most prolific mathematicians that ever lived, the Swiss Leonhard Euler (1707–1783). In 1775, when he was already blind, he published in St Petersburg, where he lived most of his life, two papers about rotations. The rotation of a sphere around its centre, for instance, is fully denoted by a line, the axis of rotation, and by an angle about that line, the angle of rotation. Euler proved that two such rotations when performed one after the other produce a third rotation, called their 'product', and as a result he described rotations in terms of three angles, called the *Euler angles*. Rodrigues (Rodrigues 1840*a*) went much further, because, given the axis and angle of each rotation, he produced a geometrical construction that determines the angle and axis of the product rotation. We illustrate this in Figure 3, where the first rotation effected is one with axis $\mathbf{a}$ and angle α, the second with axis $\mathbf{b}$ and angle β. The product rotation is around $\mathbf{c}$ by γ. Of course, it required considerable ingenuity for Rodrigues to have discovered that construction, but we refer the reader to (Altmann 1986, p. 155) for its justification. What is important is to notice that all the angles appear not as such but as *half angles*. As is often true, this apparently minor result is of momentous significance, but this was not realized until the end of Rodrigues's century: even then, it was only when quantum mechanics was developed that the half angle of a rotation became a universally used parameter.

With reference to Rodrigues's geometrical construction, it is worthwhile mentioning that Euler was never near to producing such a figure or even an algebraic analogue: like everybody else for most of the century, he used full angles and never their halves. This figure, very much used in crystallography and frequently discussed in crystallography textbooks, is however universally called the *Euler construction*, and it was, in fact, totally unknown to Rodrigues's contemporaries. Eight years after its publication, in 1848, we find the distinguished Cambridge professor of mathematics G. G. Stokes complaining that he was not aware 'of a geometrical proof anywhere published' of the result that the most general motion of a rigid body about a fixed point was a rotation. He then proceeded to produce a very clumsy discussion based on angular velocities. Accordingly, Sylvester (Sylvester 1850) rose to the challenge to produce a proper first-principles geometrical argument and proceeded independently to produce Rodrigues's construction, although, despite a figure, his reasoning is far less clear than in the work he had ignored.

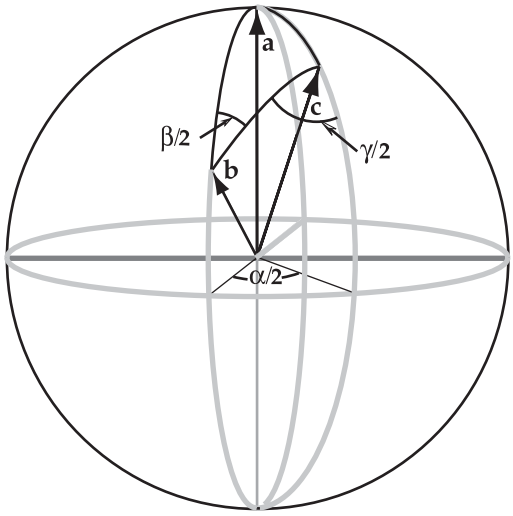

FIGURE 3. The composition or product of two rotations (so-called *Euler construction*).

Rodrigues, however, went a great deal further than his geometrical argument. By using spherical trigonometry in a straightforward manner, he was able to replace each rotation by a set of four algebraic parameters, in such a way that, given the two sets for the two successive rotations, he was able to obtain the same set of four parameters for the product rotation, a truly remarkable result. These are now the essential tool in modern work on rotations, and the four parameters that Rodrigues introduced are universally used in studying spin, in describing molecular structures, in spacecraft kinematics, in robotics, even in ophthalmology, having correctly replaced the use of the awkward Euler angles. The importance of Rodrigues's contribution, however, took some time to be properly recognized. The very successful treatise on the dynamics of rigid bodies by Routh, which reached eight editions, the first one published in 1860, contained no reference to Rodrigues until the third edition (Routh 1877, p. 176), when he proposed that the theorem on the composition of rotations be named after Olinde Rodrigues. Indeed, all the successive editions, up to the last one in 1930, contain a section labelled 'Rodrigues's Theorem.' Such recognition, however, was not readily granted. When the famous German mathematician Felix Klein published his *Vorlesungen über das Ikosaeder* in 1884, he gave Rodrigues some, soon forgotten, partial credit: he grants Rodrigues the discovery of his four parameters and acknowledges that they were unknown to Euler, but he asserts that the latter had used three analogous parameters by replacing the four of Rodrigues by the quotient of his last three by the first. This is a total misrepresentation of the facts, since, as we have said, Euler never used half angles.[29] Every opportunity was taken to bring in Euler and to push the inconvenient Rodrigues away: Schoenflies and Grübler (Schoenflies and Grübler 1902), Klein's collaborators, writing in the most prestigious mathematical encyclopedia of its time, say in relation to these formulae: '[t]hese formulae are quite often treated in connection to Euler. We owe a first essential advance to O. Rodrigues.' The

[29]See the translation of the *Vorlesungen*, (Klein 1956, p. 38).

bible of rotation theory, the monumental work by Klein and Sommerfeld, gives those four parameters without any attribution.[30] Euler's great name of course pushes Rodrigues to second place: to this day the parameters are known as the Euler-Rodrigues parameters. Worse happened to Rodrigues when this work was later discussed: Élie Cartan (Cartan 1938, p. 57), a French mathematician whose contribution to the study of rotations led the world, credits the Rodrigues paper as written by 'Olinde and Rodrigues,' a mistake copied by George Temple (Temple 1960) and repeated by others more than once.

The work of Rodrigues in 1840 is historically even more important, because it should have been used by any unprejudiced mathematician to unravel one of the most extraordinary blunders in the history of mathematics. However, because this blunder was embedded in one of the most important and ingenious creations of the century, people firmly shut their eyes to it. Sir William Rowan Hamilton, the man who in 1865 was to be ranked as the greatest living scientist by the American National Academy of Sciences, had also constructed, three years after Rodrigues, some objects composed of four elements that he named *quaternions* (see (Hamilton 1844)). He obtained them, not from a geometrical point of view, but in an a priori way through algebra. This work, from the algebraic point of view, was outstanding, but from the point of view of rotations, it was an unmitigated disaster that caused a century of confusion and strife. The reason for this is that, having discovered the quaternions algebraically, Hamilton had to interpret them geometrically. He realized of course that his quaternions were related to rotations, but he used two principles that had unfortunate consequences. The first was to give priority to algebra over geometry, whereas it was geometry, which had a strong tradition in his mathematical circle, that had allowed Rodrigues to keep his feet firmly on the ground. The second was to use only full and not half angles, for which he had very sensible reasons, which nevertheless were to be contradicted by the facts a century later. He never read Rodrigues's paper, and he never derived a product rule for the rotations as Olinde had done, but only in a different very indirect way. Cayley (Cayley 1845), who published this latter work before Hamilton did, not only acknowledged Rodrigues but confessed that he did not understand why their convoluted algebraic method gave the same results that Olinde had obtained directly, a question that was cleared up more than a hundred years later by Altmann.[31]

We shall refer the reader interested in the nature of Hamilton's blunder to the careful analysis given by Altmann,[32] which requires a great deal of detailed work, but we shall attempt here to present a rough argument to give an idea of the problem. Using modern vector notation, a quaternion is defined as a pair of a scalar a and a vector, say, $[a, A\mathbf{i} + B\mathbf{j} + C\mathbf{k}]$. A *unit quaternion* may be defined as $[a, A\mathbf{i}]$, with $a^2 + A^2$ unity. Hamilton here fell into two traps. One is that he did not realize that the vector in the quaternion is an axial and not a polar vector, an understandable failure, since he was inventing vectors at the same time that he was doing his quaternion work, and the distinction in question was not known until late in the century. It was this failure, however, that created untold problems when pursuing Hamilton's programme of defining vectors via quaternions. The highly respected mathematician Marcel Riesz (Riesz 1958, p. 21), who first pointed out

[30] See (Klein and Sommerfeld 1897, Vol. 1, p. 56 and Vol. 4, pp. 939–944).
[31] See (Altmann 1986, p. 16) and (Altmann 1989).
[32] See (Altmann 1986, Chapters 1 and 12), (Altmann 1989), and (Altmann 1992, Chapter 2).

this error, qualified Hamilton's interpretation as 'grossly incorrect.' The second trap is that Hamilton noticed that a quaternion $[\cos\alpha, \sin\alpha\mathbf{i}]$ acting on a vector normal to $\mathbf{i}$ 'rotates' it by the angle α and interpreted this as a rotation by α around the axis $\mathbf{i}$ of any vector normal to $\mathbf{i}$. This, however, is not at all a rotation of the vector, but an artifact resulting from the product of two rotations, as was demonstrated by Altmann (Altmann 1989, p. 304). It was clear that there was something wrong in Hamilton's interpretation, since the same so-called 'rotation' acting on a general vector does not transform it into its rotated vector, a result that of course Hamilton knew, but preferred to ignore. In fact, he often took the primary definition of a quaternion as an operator that acting on a vector rotates it into another vector or, what is the same, as the quotient of two vectors. This erroneous statement still appears in the mathematical literature as, for instance, in (Hankins 1980, p. 311) and is the definition that was used by the highly respected Oxford dictionaries until, at Altmann's instigation and with the approval of the chairman of the dictionaries' mathematical committee, Sir Michael Atiyah, it was changed. Even now, it is still given in the current edition of the *Oxford English Dictionary*, although it will soon be changed.[33]

Rodrigues's paper goes much further than we have described. When symmetry operations are combined, their set forms what mathematically is called a *group*. Rodrigues considered not only the rotation but also the translation group, the aggregate of which forms the so-called *Euclidean group*. Translations appear to be totally distinct from rotations, but Rodrigues realized them by producing infinitesimal rotations (rotations by tiny angles) around infinitely distant rotation axes. It was to take more than half a century before this remarkably clever idea was again used. Meanwhile, as is clear by now, Rodrigues's work was largely ignored until its great importance was rediscovered by the mathematical historian Jeremy Gray (Gray 1980), who produced a very penetrating analysis of his article. (See Chapter 8.)

As we have seen, even at this time Rodrigues had not totally abandoned mathematics, and he still read the mathematical literature. He had done nothing along the lines of this paper, however, since 1814, and it is a wonder why he returned to the subject. It must be remembered that geometry had been a strong influence in his mathematical upbringing: Monge had created a powerful school, of which Michel Chasles was an important member. A school friend of Rodrigues, Chasles had continued their acquaintance: we know that they met at the salon of the Saint-Simonians in the 1830s (Bertrand 1902, p. 40), and it is possible that his influence

[33]Hamilton's use of full rather than half angles, as in the quaternion $[\cos\alpha, \sin\alpha\mathbf{i}]$, entails another very important mistake. Given that quaternion form, he took the unit quaternion $\mathbf{I}$, $[0, \mathbf{i}]$ to be a rotation by $\pi/2$ around the axis $\mathbf{i}$. On defining the unit quaternions $\mathbf{J}$ and $\mathbf{K}$ in relation to directions $\mathbf{j}$ and $\mathbf{k}$ orthogonal to $\mathbf{I}$, he then interpreted the relation $\mathbf{IJ} = \mathbf{K}$ as a rotation by $\pi/2$ of $\mathbf{j}$ around the axis $\mathbf{i}$ that transforms $\mathbf{j}$ into $\mathbf{k}$. Although this interpretation is demonstrably wrong (see the correct multiplication rules in (Altmann and Herzig 1994, p. 598), it still is most often stated as the meaning of the quaternion units (see, for instance, (O'Donnell 1983, p. 143)). It can immediately be seen that, on using Rodrigues's parametrization in half angles, the quaternion given should be written as $[\cos(\alpha/2), \sin(\alpha/2)\mathbf{i}]$, so that $\mathbf{I}$, $[0, \mathbf{i}]$ corresponds now to a rotation by π (*binary rotation*) and not by $\pi/2$. Therefore, the equation $\mathbf{IJ} = \mathbf{K}$ really means, following Rodrigues, that a binary rotation around $\mathbf{i}$ times a binary rotation around $\mathbf{j}$ equals a binary rotation around $\mathbf{k}$. This relation is true beyond any possible doubt (see (Altmann 1986, Table **2**-5.2, p. 49)); and it is so fundamental that if it were untrue, the whole of the science of crystallography would collapse.

revived Rodrigues's thoughts on rotations. It is important to remember that precisely at that time, in 1830, Chasles produced one of the fundamental results in mechanics, called to this day the Chasles theorem, namely, that the most general motion of a rigid body is a translation combined with a rotation around a fixed point. It is highly probable, therefore, that this problem was discussed by the two friends and that it remained in Rodrigues's mind.

Three years after the seminal 1840 paper, Rodrigues produced his final cluster of three mathematical papers (Rodrigues 1843*a*, 1843*b* and 1845). The first two are concerned with trigonometrical problems and are followed in 1845 by a short note on continued fractions, his last mathematical paper.

Before we leave this brief discussion of Rodrigues's mathematical works, it is important to mention his writing style. No one who has read the original works of the mathematicians of his period, however great, can help experiencing a sense of relief in reading Rodrigues. His language is concise and entirely free from jargon, and his form of exposition is amazingly clear. Amongst those who have had that experience, the word 'gem' often recurs. The contrast with Hamilton, in particular, is staggering, and one can see why (even (Hankins 1980, p. 322) qualifies Hamilton's *Lectures* as 'unreadable'). Hamilton wrote sometimes starting from preconceptions, and when these proved a trap, he refused to abandon them.[34] Preconceptions were never part of Rodrigues's mathematical writing, which could thus flow from fact to fact without impediment. The opposition of French rationalism with the *Naturphilosophisch* quasimysticism of Hamilton and his followers (it is not without significance that Hamilton was a friend of the poet Samuel Coleridge) could not have a better example than the comparison of these two authors.

Olinde Rodrigues: Banker, social reformer, Saint-Simonian

We have already seen that when it was clear that the Restoration had destroyed any chances he might have had of an academic career, Rodrigues joined forces with Myrtil Maas and worked on producing actuarial tables for insurance companies. The collaboration did not stop there, as their pamphlet (Rodrigues and Maas 1820) on the '*caisses hypothécaires*' (mortgage banks) shows. This exemplifies two interests that were central to Olinde for the rest of his life: financing and how to use it for social purposes. Following his father, he became a freelance broker at the Bourse ('kerb-broker'); soon, in 1823, he was the director of the *Caisse Hypothécaire* at rue Neuve-St-Augustin,[35] and in 1825 he contributed to a pamphlet on banking (Rodrigues 1825). He may have continued for a period to teach mathematics as a private tutor at the *Polytechnique*, but after 1825 there are no further records of any such activities. In 1828 he became managing director of the *Caisse Hypothécaire*, where he would appoint his former pupil Prosper Enfantin general cashier. Probably, his detachment from mathematics in this period is due not only to his banking interests but also to his involvement with the philosopher

[34] As already mentioned, Hamilton first obtained his relation between quaternions and rotations of vectors by considering the rotation of a vector normal to the axis of the quaternion, and although this approach broke down for a vector in a general orientation, he never abandoned his original point of view. (See (Altmann 1986, Chapter 12) and (Altmann 1989, p. 304).) In the latter reference it will be seen, in fact, that Hamilton's original example was not a rotation of the given vector at all, but a product of two rotations that by a freak appears to do what Hamilton claimed.

[35] See (Booth 1871, p. 107) and (d'Allemagne 1930, p. 29).

Saint-Simon and, following the latter's death in 1825, with the movement that came to be known as Saint-Simonism, which was going to be a central part of his life, and the one for which he is best known in the history books on this period.

Claude-Henri de Rouvroy, comte de Saint-Simon, second cousin of the famous Duc de Saint-Simon, was born in 1760, fought alongside Washington in the American war for independence, and not only survived the Terror on his return, but became wealthy through land speculation as a result of it. At the age of 38 he studied mathematical physics at the *École Polytechnique*, Monge, Lagrange, D'Alembert being not only his teachers but also his friends. He had visions of a lay religion, and he proposed at the time the public worship of Sir Isaac Newton. More practically, he devoted himself to numerous proposals for the reorganization of society. A charismatic figure, Saint-Simon inspired his followers with a quasireligious fervour, but his life was very irregular, and he soon went through his money and fell into debt. After the restoration of the Bourbons in 1815 he gradually became more isolated, felt betrayed by his friends, and very systematically, on 9 March 1823, prepared to commit suicide, loading his pistol with seven large-calibre bullets.[36] He then placed his watch on his desk and wrote his last thoughts until the allotted time arrived. His aim, alas, was so poor that of the seven shots to his head only one penetrated his cranium, causing him to lose one eye but not his life (Manuel 1962, p. 113). He was cured in two weeks, although of course he was left in very poor health.

A couple of months later, to alleviate Saint-Simon's illness and destitution, the banker Ardoin, who had already in January introduced his friend Rodrigues to the count, brought him to meet the invalid (also later Léon Halévy, brother of the composer Fromental Halévy) in the hope that the well-off Olinde would provide support.[37] This Rodrigues did and to a very large extent: right from the beginning he took the place of the philosopher Auguste Comte in collaborating with Saint-Simon, and during the next two years until the latter's death he was his constant companion, carefully recording his thoughts. Not only did he support Saint-Simon financially, but he also helped the master finish his last work, *Nouveau Christianisme*, and financed its publication shortly before his death. Thus, he was far more than an amanuensis for Saint-Simon: without him there would never have been a Saint-Simonian movement. On 19 May 1825,[38] Saint-Simon died in Rodrigues's arms, placing on him the mantle of leadership of the Saint-Simonians with the words: '[t]he pear is ripe: you must pick it' (Weill 1894, p. 32). It was then that Saint-Simonism took off as a visionary social philosophy with undertones of religious mysticism. Olinde's brother Eugène, then a brilliant young student at the University of Paris, played a part in proselytizing for the new faith with infectious enthusiasm. He also devised a hierarchy for the membership of the movement, along the lines of a religious community: the leaders, Barthélémy-Prosper Enfantin—who had been Olinde's mathematical pupil when at the *Polytechnique* and later his cashier—Saint-Amand Bazard, and Rodrigues, were called *Pères*, and they in turn addressed the brethren as *Fils*. But it was Rodrigues who provided help, finance, and advice to the movement, and there was much more: for Olinde, as had been true for Saint-Simon, it was important to introduce the aesthetics of romanticism into

[36] See (Weill 1894, p. 30) and (Muso 1999, p. 18).
[37] See (d'Allemagne 1930, p. 29) and (Weill 1894, p. 30).
[38] 12 May for (Coignet 1883).

the old Christianity. Later, Enfantin was to say, addressing the Saint-Simonians, '[y]our father Rodrigues was the only one who constantly repeated to us that this book [Saint-Simon's *Nouveau Christianisme*] had enclosed within it the most lofty teaching which was ever given to man to receive' (Manuel 1962, p. 144).

We shall give only that account of the principles of Saint-Simonism[39] that is necessary to understand the circumstances surrounding Rodrigues's life. The New Christianity preached by Saint-Simon was not only an early form of socialism but also a fully fledged universal and lay religion, in which, as noted, even a cult of Sir Isaac Newton was proposed (Muso 1999, p. 86). It must be understood, however, that this was not a mere rationalist approach. The prophet had even said 'God has spoken to me,' and when later a proper cult was formed, a rigid discipline was expected, ruled by the *Pères* and strictly followed by the faithful.

By 1825, with Enfantin's help, Rodrigues founded a journal, *Le Producteur, Journal de l'Industrie, des Sciences et des Beaux Arts*, of which Bazard was also an editor, shares of 1,000 francs each being issued to fund the publication (Charléty 1931, p. 30). It started publication as a weekly on 1 October 1825, but it became a monthly on 1 April 1826, and it ceased publication altogether six months later. Although the paper lasted little more than a year, it helped to unify the group of supporters, and soon the liberal daily *Le Globe*, although more independent of the Saint-Simonians, helped propagate their views, until it was taken over by them in November 1830, when it became the *Journal de la Doctrine de Saint-Simon*. This was largely supported by donations from the Saint-Simonians, although it lasted only until 20 April 1832 (Muso 1999, p. 101.)

The meetings of the community, full of religiosity and excitement, first took place at Rodrigues's *Caisse Hypothécaire* on rue Neuve-St-Augustin, then in salons in Enfantin's apartment at 6 rue Monsigny and later in his house on the outskirts of Paris (now the 20*ème arrondissement*) in Ménilmontant. During this period Rodrigues published three works (Rodrigues 1829, 1831*a*, and 1831*b*), of which the last two are his appeal to the Saint-Simonians at a meeting held in the Salle Tailbout on 27 November 1831 and published in *Le Globe* of the 28th. In it Rodrigues asks all poets to 'sing the hope of a hardworking people which wishes no more to make war.' A new hymn to peace was required to replace the *Marseillaise*, which was a war song. Painters and sculptors were also encouraged to spread the new ideals, and appropriate music had to be forthcoming to surpass the works of Rossini and Beethoven, a proposal to which Mendelssohn took offence (Locke 1986, p. 62).

Enfantin, although he had met Saint-Simon only once (Charléty 1931, p. 65), was not happy as a mere member of the triumvirate ruling the Saint-Simonians and wanted to become their unchallenged leader. His aims, also, were more extreme than those of Rodrigues: not only the founding of a new religion but the very destruction of Christianity.[40] On Christmas 1829 the faithful elected him and Bazard as *Pères* of the *Famille Nouvelle*, and on 31 December Rodrigues abdicated, feeling that his own mission had been accomplished (Courteault 1925, pp. 156–157). His decision might have been influenced by the rapidly declining health of his much loved brother Eugène, who died a few weeks later on 13 January 1830, leaving him disconsolate. This did not prevent him manning the barricades on July 27, 28, and

[39] A concise and clear discussion of Saint-Simonism may be found in (Muso 1999).

[40] See (*École Polytechnique* 1895, Vol. 3, p. 489).

29, when the population of Paris rebelled against the Bourbons and drove King Charles X into exile.

Towards the end of 1831 relations within the community became tense. Enfantin held that he, as a *Père*, was entitled to have sex with the women of the sect (d'Allemagne 1930, p. 217), and he even wrote a letter to Claire Bazard, the wife of Saint-Amand Bazard, as active a leader of the community's women as she was good looking, demanding that she find him a female companion.[41] (Enfantin, in total contrast with Rodrigues, was very charismatic, extremely handsome, and extremely convinced that this was the case.) Not unnaturally, on 11 November 1831 Bazard, and with him nineteen other dissidents, left the community, and very much to his satisfaction, Enfantin became *Père Suprême*. It is surprising that at this stage, despite fundamental disagreements with Enfantin, of which we will say more later, Rodrigues remained loyal to the master, who on 27 November declared a new cult founded that was even more hierarchical. On the next day the Saint-Simonians signed the necessary papers that made them collectively responsible for the group's now perilous finances and gave Olinde power of attorney on all their assets (Charléty 1931, p. 152). In January 1832 Enfantin was confirmed as *Père Suprême*, with Rodrigues as *chef du culte*. This situation did not last long. Admittedly, Rodrigues had become almost a slave of the Bourse, his attendance there being essential for his business, but there were far more serious issues. He strongly objected to Enfantin's outlandish views on sexual morality, which claimed incest and adultery as ingredients of the new moral code. On 12 January 1832, in an article by Duveyrier based on Enfantin's ideas, *Le Globe* announced the impending coming of the *Femme-Messie*, whose arrival was expected to herald a vision of life as a banquet of delights, including free choice of sexual partners.

Already, when early in 1830 Enfantin presented his projected moral code to Rodrigues and Bazard, Olinde qualified this in a letter as a monstrous heresy (d'Allemagne 1930, p. 215). On 17 October 1831 he had presented to the Saint-Simonians counterproposals for a moral code (d'Allemagne 1930, p. 221), but in the power struggle that ensued, Enfantin was not averse to stooping to strike Rodrigues at what the latter held most dear. For Rodrigues, his fifteen-year-old marriage to Euphrasie was central to his life and to his principles, whereas Enfantin relished telling him that his wife had asked him, Enfantin, for help to get away from her husband.[42] This, for all we know, could have been a complete fabrication, Enfantin not being the most veracious of men. All this had become too much for Olinde, and in February 1832 he decided to leave the movement. The break was dolorous: he was expelled from his own apartment on rue Monsigny, for the maintenance of which he had paid 200 francs per annum;[43] in his turn, of course, he blocked all the financial records of the Saint-Simonians (Ratcliffe 1971, pp. 1229–1230). In March, as a final

[41] See (d'Allemagne 1930, p. 216). That something more drastic may have happened between the two might be inferred from a later (1832) letter of Claire Bazard to Cécile Fournel, in which she equates Enfantin with Satan (d'Allemagne 1930, p. 239).

[42] See (d'Allemagne 1930, p. 243). This problem is thoroughly discussed by Paola Ferruta in Chapter 4.

[43] Personal communication from the files of Dr. Jacques Béjot, Paris.

attempt to save the movement, Rodrigues proclaimed himself the sole head of the Saint-Simonians, but nothing much came of this.

Their affairs, in any case, were not going well because Louis-Philippe's government started applying pressure on opponents of the regime, and charges were brought in 1832 against several prominent Saint-Simonians, Enfantin and Rodrigues amongst them, on the grounds of outrages against public morality, Enfantin in particular having been entirely open in his support for total sexual freedom. (Moreover, they were accused of illegal association, meetings of more than twenty people being prohibited by Art. 291 of the *Code Pénal*.) The trial was bizarre, Enfantin indulging in exhibitionism, dressed in flamboyant, very lightly coloured clothes, with '*le Père*' embroidered upon his chest. During his deposition he paused from time to time and silently gazed at the judges for minutes, asserting when challenged that 'they would come under the influence exercised by his appearance.... The Attorney General had yet to learn the full power of beauty' (Booth 1871, p. 189). This was all to no avail, since Enfantin was sent to the *Sainte-Pélagie* prison for a year (after which he abandoned public life and became a hard-headed businessman). Rodrigues escaped more lightly, with a nominal fine of 50 francs. (An account of this trial is given in (Rodrigues 1832*b*, 1832*d*).) Three other pamphlets on Saint-Simonism (Rodrigues 1832*a*, 1832*c*, 1832*e*) were also published in this period. The first is particularly interesting and had been published in *Le Globe* on 19 February 1832 as a manifesto on his proposed moral code, which reveals Rodrigues's serious interest in promoting the status of women. This article is in fact a major milestone in the history of feminism.

Despite the schism, Rodrigues remained a Saint-Simonian at heart for the rest of his life: the words told to him by Saint-Simon on his death bed, 'Rodrigues, you must not forget, and remember also that in order to do great things it is necessary to be passionate' (reported in *Le Globe* 30 December 1831 (Charléty 1931, p. 23)) were engraved like a motto in his mind. In 1832, however, he published his last works as a Saint-Simonian, (Rodrigues 1832*e*), and an edition of the works of the master (Rodrigues 1832*f*). Later, he produced two more compilations of Saint-Simon's works: (Rodrigues 1841*b*) and (Rodrigues 1848*a*).

As a banker, Rodrigues understood the importance of securing capital investment and his views, and those of others, contributed to the foundation of new financial institutions, in which the Pereires took a major part (Ratcliffe 1972). A bill to renew the charter of the Bank of France caused Rodrigues to make his own proposals (Rodrigues 1840*b*). Rodrigues held that transport was an engine of social improvement, and some of the first French railways, which played such a pivotal role in France's industrialization, were partly a result of the efforts of several Saint-Simonians. In 1835 a cousin (and brother-in-law) of Rodrigues, Émile Pereire, gained the concession from the government to build the first passenger line, from Paris to St-Germain. Clapeyron and Lamé, old school friends of Rodrigues and marginal Saint-Simonians, were responsible for the construction, Clapeyron himself being in charge of locomotive design (Ratcliffe 1972). In 1846 Enfantin, now settled down as an active entrepreneur (d'Allemagne 1935), became the director of the three railway companies that covered the Paris-Marseille route, as well as

secretary general of the Paris-Lyons line. He also formed a group to draw a plan for a Suez Canal and even one for a Channel tunnel.[44]

In 1840 Rodrigues founded a journal, *Le Patriote de 1840*, to address the gulf between the workers and the new but by now well-established bourgeoisie, and he also published a pamphlet on pacifism (Rodrigues 1840*c*). This journal soon closed down, but the contact that Rodrigues had formed with the workers led him to publish a 500-page collection of workers' poetry, *Poésies sociales des ouvriers* (Rodrigues 1841*a*), which was followed by Rodrigues (1841*c*). There was now a hiatus in his public life that lasted seven years, until the 1848 Revolution brought a revival of the Saint-Simonian spirit and inspired Rodrigues to draw up a project for a constitution of the Republic and another for universal suffrage (Rodrigues 1848*b*, 1848*c*). Although of course they were not adopted, they had a significant influence in furthering the rights of both women and workers and in creating an interest in elections. With his Saint-Simonian friend Gustave d'Eichthal (1804–1886), Rodrigues fought for the abolition of slavery, which finally came about in 1848 (Benbassa 1999, p. 121). He even proposed the establishment of working men's holidays, so novel an idea that it was not adopted until 1912. His concerns about the organization of labour are also reflected in (Rodrigues 1848*c*) and (Rodrigues 1848*e*). One of his last works, (Rodrigues 1848*d*), was about banking, and again contained his radical critique of the existing structures of the Bank of France.[45]

Saint-Simonians and Rodrigues himself were not only involved in social and political matters but also exerted a profound influence on the cultural life of France in this period. Although their incursions into music were not always welcome (Mendelssohn resented Olinde's efforts to convert him, as discussed by Locke),[46] they left their mark, for instance on the works of Fromental Halévy. Franz Liszt, who was introduced to Saint-Simonism in 1830, was attracted to the idea of a community based on brotherly love as described in Rodrigues's *Réunion de la famille* (Rodrigues 1831*a*), which he had read with approval and which he commended to George Sand (Locke 1986, p. 101). From his youth Rodrigues had been passionate about music: while he was a *maître d'études* in 1814–1815 at the *Lycée Napoléon*, not only did he teach Charles Lambert mathematics, but he kindled his passion for music.[47] It is probable that this mutual interest was one thing that attracted him to his young bride Euphrasie.[48] He also took an active role in the musical activities of the community, singing lustily at their meetings, and he frequented musicians. Enfantin, for instance, met Mendelssohn at his home (Locke 1986, p. 334).

We do not know much about Rodrigues's last few years except that he of course was much affected by the Revolution of February 1848, which brought him back to libertarian ideas. Also, his social sensitivity was outraged at the high unemployment that the flight of the well-to-do bourgeoisie (and of their capital) had exacerbated.

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[44]See (Pinet 1894), (Charléty 1931, p. 285), (Walch 1970*a* p. 624), and, especially, (d'Allemagne 1935) and (Walch 1970*b*), where the influence of the Saint-Simonians on industrial developments is fully discussed. Barrie Ratcliffe (1995), however, believes that this influence has been exaggerated.

[45]For lists of these publications by Rodrigues we have consulted (Walch 1967) and (Gerits 1986).

[46]See (Locke 1986, p. 108).

[47]Unpublished notes from the ex-Saint-Simonian Charles Lambert bequeathed to the *Bibliothèque de l'Arsenal*.

[48]See Chapter 4 by Paola Ferruta.

A meeting of old Saint-Simonians was summoned at Rodrigues's house with a view to regenerating the movement, but with the exception of Isaac Pereire, no one agreed to this (Locke 1986, p. 200). Yet, Rodrigues's idealism was not dead, and he went to London, where the Chartists, in sympathy with the Revolution in France, planned a large public demonstration for 10 April 1848. On the eve of that day he was seen near the entrance to Somerset House in the Strand getting up on a chair to harangue the crowd. A policeman told him to hold his tongue and get down, which he refused to do, whereupon the officer pulled him down and hit him with his truncheon (Reid 1902, p. 132). A few years later Rodrigues died in his modest apartment on rue d'Amsterdam (in the present-day 8*ème arrondissement*) largely forgotten (Michaud 1843, pp. 288–289). Even the date of his death is often misquoted in the literature: 26 December 1850 in Michaud.[49] The records at the Paris cemetery of *Père-Lachaise*, where he is buried near Saint-Simon's tomb, show unequivocally, however, that he died on Wednesday 17 December 1851, that his wife Euphrasie signed the death certificate, and that he was interred on the 19th: even as a long-lapsed Jew, the Jewish tradition of early burial had been respected. We know that he died in the arms of Isaac Pereire, and it is said that his death was the result of a minor accident (Ratcliffe 1971, p. 1231). On the death of his wife Euphrasie, she was interred in Olinde's tomb, a single headstone commemorating both of them.

As we have seen, history was not very kind to Rodrigues. Most of the short biographical notes on him make no reference to his being a mathematician. Michaud (Michaud 1843) calls him an 'economist and social reformer'; for Courteault (Courteault 1925) he is only a Saint-Simonian. Both Benbassa and Cavignac refer to him, erroneously, as the 'financier of Saint-Simonism.'[50] Booth (Booth 1871) calls him *Rodrigue* throughout his book, as is also the case for Coignet (Coignet 1883), whereas *Rodriques* is the way by which he is known to Edmund Wilson (Wilson 1941, p. 100). Not too long ago, a distinguished professor at the *Collège de France* referred to him as *Olindes* Rodrigues (Chambre 1970). As for the mathematicians, their nominal travesty, as already recorded, is even worse.

Olinde Rodrigues: The man

Many books on Saint-Simonism include pictures of the main protagonists, but we have found none with a portrait or sketch of Rodrigues, even in the monumental and well-illustrated tome of d'Allemagne (d'Allemagne 1930). Until the present time the only picture extant was that given by Courteault (Courteault 1925, p. 157). Fortunately during the preparation of this book three more pictures have been discovered, and they are shown here, together with the one in Courteault's article. Plate 1, which represents Rodrigues as a fairly young man, c. 1830, was found in the Archives of the Pereire family in Paris. Plate 2, the only portrait so far published, appeared in (Courteault 1925, p. 157 and also the frontispiece on p. 152), and it is a reproduction of a portrait that at the time of that publication was in the possession of Mme. Deutsch de la Meurthe.[51] This portrait shows Rodrigues probably in his

[49]See (Michaud 1843, pp. 288–289). This date is also given by Cavignac (Cavignac 1987), although we have not been able to find the origin of this error.

[50]See (Benbassa 1999, p. 198) and (Cavignac 1991, p. 357).

[51]Georgette Deutsch de la Meurthe married (1918) Gaston Gradis, a grandson of Henri Gradis (1823–1905), himself a grandson of Laure Sarah Rodrigues-Henriques, a member of the 'Abraham' branch of this family. M. Henri Gradis, son of Georgette, has kindly informed us that

late thirties or early forties. Plate 3, part of the Pereire family archive, is likely to be a lithograph from around 1840, and it is marked Imp[rimerie] Lemercier & C^{ie}. Plate 4, instead, must be of Rodrigues at a later date. It was discovered by Dr. Paola Ferruta in the *Fonds Eichthal* at the *Bibliothèque de l'Arsenal*, Paris, and is a photograph by the famous photographer Félix Tournachon Nadar (1820–1910), dated 1887 (36 years after Rodrigues's death) and signed 'Nadar'.[52]

As for Rodrigues's physical appearance and character, hardly anything is ever mentioned except for the references in (Coignet 1883). The author of this paper is almost certainly the French historian Clarisse Coignet, born in 1823, who writes as one who knew Rodrigues personally, which is highly probable. Coignet (Coignet 1883, pp. 131–132) describes Rodrigues as small and bony, without being thin, having a regular face with his hair and sideburns curly. His walk was uneven and quick, his talk brief and brusque, sometimes with prophetic accents. As for his character, the evidence appears somewhat contradictory and has to be carefully assessed. Undoubtedly, he suffered in comparison with the charismatic Enfantin: Michel Chevalier compares his authoritarian manner with the 'great magnetizer' that Enfantin was (Ratcliffe 1971, p. 1226). There is an anecdote repeated more than once[53] that makes him appear in the wrong light. In one of the Saint-Simonian meetings it is said that Rodrigues insisted on a testimony from everyone present that the Holy Ghost dwelt within him, Rodrigues, and that when someone expressed disbelief, he fell down in a fit, whereupon a doctor had to be called; but by the time he arrived, Rodrigues was fully recovered, the quarrel having been resolved. This episode, however, has to be interpreted with an understanding of what Saint-Simonian meetings were like. In fact, Coignet (Coignet 1883, p. 159) makes it quite clear that this meeting, in October 1831 on rue Monsigny, came as a result of Enfantin requiring the faithful to submit to total and public confession, no personal trait, however intimate, being beyond discussion. Such meetings lasted sometimes ten hours at a stretch and such was the strain that even young people were subject to fainting during them. Rodrigues, as '*chef du culte*' was acting entirely within Saint-Simonian doctrine when demanding a statement of faith: the priest was supposed to reveal the law and to determine its application by virtue of an afflatus exclusive to him (Coignet 1883, pp. 153–154). Furthermore, if Rodrigues's demand appears excessive, compare it with Enfantin: 'I am Man-God,' stated in a letter to Duveyrier of 21 June 1830 (d'Allemagne 1930, p. 252). Coignet writes that it was the doctor who required a retraction from the doubter, Jean Reynaud, in order to save his patient, who soon recovered, and that such was the pathos of this meeting that a number of other members remained ill for several days (Coignet 1883, p. 159).

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the present whereabouts of this portrait are unknown. We are grateful to M. Paul M. Siméon for the genealogical information in this note.

[52]This picture must be a photographic reproduction of a daguerreotype or perhaps an engraving done in the 1840s. Fortunately, other known examples exist where records show that Nadar, despite his great fame, used to engage in such practice (personal communication from Gordon Baldwin, of the Getty Museum in California, and Barbara Tannenbaum, Chief Curator of the Akron Art Museum in Ohio). It can safely be assumed, therefore, that this is indeed a reproduction of an earlier picture done in Rodrigues's lifetime. Mme. Françoise Reynaud, Curator of Photography at the *Musée Carnavalet*, Paris, suggests that the original, rather than a daguerreotype, was an engraving, or a lithograph, of which unfortunately there is no trace. Similarities in style and dress with Plate 3 suggest that the original was by the same hand.

[53]See (Weill 1894, p. 102) and (Ratcliffe 1971, p. 1230).

PLATE 1. C. 1830, from the *Archives de la famille Pereire*, Paris.
By kind permission of Mme. Geraldine Pereire-Henochsberg.

REVUE PHILOMATHIQUE

OCTOBRE - DÉCEMBRE 1925.

Olinde Rodrigues
d'après un portrait de famille.

Plate 2. Frontispice on p. 152 of the article by P. Courteault (Courteault 1925). Un bordelais Saint-Simonien. *Revue Philomatique de Bordeaux et du Sud-Ouest* **28**, kindly supplied by the *Bibliothèque des Archives Départamentales de la Gironde*, Bordeaux.

PLATE 3. C. 1840, from the *Archives de la famille Pereire*, Paris.
By kind permission of Mme. Geraldine Pereire-Henochsberg.

PLATE 4. From *Bibliothèque de l'Arsenal*, Fonds Eichthal, XXXII, manuscript 14110. This photograph is dated 1887 and signed by [Félix Tournachon] Nadar. Reproduced by permission of the *Bibliothèque Nationale de France*.

It is likely that Rodrigues's character, as a man of severe principles and strong opinions, was not an easy one. Claims have been made that Rodrigues did not play a major role in the Saint-Simonian movement because of his irascible character, but such a trait is not mentioned by Coignet. There is a letter from Michel Chevalier to Rodrigues, where he accuses the latter of having the speech and gestures of an 'army commandant,' and this expression is sometimes taken against Olinde. It must be remembered, however, that this is the editor of *Le Globe* with whom Rodrigues had serious doctrinal differences and to whom he addressed an open letter (Rodrigues 1832*c*). Although these words are often quoted, the actual expression used by Chevalier is not quite as harsh: 'commandant of a pacific army of workers,' as can be seen in Chapter 4 by Paola Ferruta, where Chevalier's letter is treated in detail.[54] It must not be forgotten either that the Saint-Simonians were an unruly and argumentative lot and that their behaviour could be totally outrageous—often bound to disturb any sensible person, let alone Rodrigues. That he tried to placate discord is an undoubted fact, acknowledged even by Enfantin at a time when he was no friend of his. Writing from prison, he said: 'I would like to speak of the peaceful sentiment that Rodrigues finally managed to inculcate in us' (d'Allemagne 1930, p. 72). Also, d'Allemagne (d'Allemagne 1930, pp. 124–125) quotes opinions on Rodrigues's insistence on 'peaceful words,' and 'words of peace.' He could be extremely patient, even when dealing with ideas that he did not like: when at a meeting in 1831 Jean Reynaud attacked the morality of Enfantin's ideas on women, he spent one and a half hours calming him down (Locke 1986, p. 91). On the other hand, he was intense in defending the ideas of Saint-Simonism, and he could be tedious in proselytizing for them, which sometimes put people's backs up, as was the case with Mendelssohn.

The most detailed study of Rodrigues's character is given by Coignet (Coignet 1883, pp. 131–145) who shows him as sincerely disinterested personally and totally devoid of egotism. On the other hand, as a religious puritan, he was sometimes prepared to sacrifice individual rights and freedom. 'He spoke little and never from his own authority,' and he was capable of the greatest sacrifices. During the debates at the end of 1829 he showed a rare and total selflessness in his own personal position. Courteault (Courteault 1925, p. 166) gives a favourable sketch of Olinde's character: for him, he was a Jew who 'practiced personally the old Christian virtues of disinterest and charity.' 'He was good, of a candid goodness and of a naive generosity,' a believer in 'the natural goodness of men.' This is of course the view of one who judged Rodrigues by his deeds: some of the Saint-Simonians who opposed him may have seen a different side to his character. Letters from women to *Le Globe* often praise his character. Francisca Prugniaux, for instance, lamenting his departure from the Saint-Simonian church in 1832, wrote: 'It was he [Rodrigues] who has given me much to think about, who has inspired me with ideas which have put me on the road of progress. It is thanks to him that I overcame the prejudices that I have entertained all my life against Jews.'[55] Despite all his manifold activities, Rodrigues was a staunch family man. We have already seen the influence he had on his younger brother Eugène, but he also introduced his cousins,

[54]It is worthwhile mentioning that Chevalier's expression is not metaphorical: in (d'Allemagne 1930, p. 364), there is a whole section on the marches organized for the '[*a*]*rmée pacifique des travailleurs*.'

[55]Letter published in *Le Globe* on 27 February 1832. See (Riot-Sarcey 1998, pp. 126–127).

the Pereire brothers, who later became very influential figures in France,[56] to the sophisticated intellectual and banking circles that frequented his household.

Rodrigues is sometimes presented as some sort of a failure because it was clear that he lacked Enfantin's charisma and had thus to yield the leadership of the Saint-Simonians to him. It must be understood, however, that by the time Rodrigues separated from the movement, it had become a totally discredited group, ridiculed by most French intellectuals. It is totally unrealistic to imagine that Rodrigues would have wanted to head such a group: Enfantin's 'victory' was nothing more than the destruction of what could have been a respected force in France's social development. Historians of the French labour movement recognize this: in his comprehensive dictionary of this movement J. Maitron writes: 'More than many other Saint-Simonians, Olinde Rodrigues kept faith with Saint-Simon's desire to improve the fate of the poorest and of the largest number of people.'[57]

We might speculate as to what Rodrigues might have produced had he been able to develop as a full-time mathematician, but as a banker, a financier, and a Saint-Simonian, he prepared the foundations for the just social treatment of workers, for the abolition of slavery, for the rights of women, and for controlling the power of capital, all ideas that informed early socialism. His few mathematical works are of such quality that they alone should ensure that he has a major place in the mathematical pantheon.

Acknowledgements

The authors are grateful to M. Jean Savignac, Mme. Claudine Billoux, and the Secretary of the *Association des anciens élèves de l'École Normale Supérieure* for useful correspondence. Also, the authors thank Dr. Paola Ferruta for details of Rodrigues's marriage certificate and most especially for generously providing us with the Rodrigues portrait in the *Bibliotèque de l'Arsenal*. Professor Jeremy Gray kindly drew our attention to R. P. Locke's book. Mme. Stéphane Toublanc, of the *Cercle de Généalogie Juive* of Paris, gave us very useful information about the Decree of Bayonne, for which we are grateful, as well as for her helpful and patient treatment of our numerous inquiries. The first author is extremely grateful to Professor Paulo Almeida of Lisbon for a very useful discussion on Olinde Rodrigues's works on differential geometry.

M. Louis Bergès, director of the *Archives Départementales de La Gironde* in Bordeaux, has very kindly provided us with a photographic reproduction of the Rodrigues portrait published in 1925 by Courteault in the *Revue Philomatique de Bordeaux et du Sud-Ouest*. We are grateful to Gordon Baldwin, of the Getty Museum in California, and Barbara Tannenbaum, Chief Curator of the Akron Art Museum in Ohio, for information about Felix Nadar, and we are much indebted to Arthur Ollman, Director of the California Museum of Photographic Arts, for putting us in touch with these scholars. Also, we thank Mme. Françoise Reynaud, of the Carnavalet Museum, Paris, for a discussion about the Nadar photograph. Mme. Stéphane Toublanc of the *Cercle de Généalogie Juive* in Paris helped us very much over the years in the use of some of the Paris Archives, and we would like to record with gratitude her kind support and the extremely useful advice that she has

[56] See (Ratcliffe 1971), (Ratcliffe 1972), (Autin 1984).

[57] See (Maitron 1966, p. 332). This biographical article on Rodrigues never mentions his being a mathematician.

given us personally and by correspondence. We are also most grateful to Dr. Jacques Béjot, a member by marriage of the Pereire family, who most generously provided us, through Dr. Paola Ferruta, with large files of genealogical notes on the Pereire-Rodrigues family, prepared by Dr. Patrice Assouad and other Parisian genealogists, some of them members of the Rodrigues-Henriques and Pereire families. We are also grateful to M. Paul Siméon and Dr. Patrice Assouad for discussion on these results.

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These works are listed in chronological order, including those with collaborators. The dates of mathematical works are given in bold.

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(1815*a*) *De l'attraction des sphéroïdes: thèse soutenue devant la Faculté de Sciences de Paris par M. Rodrigues, docteur-ès-lettres* [sic], *le 28 juin, 1815*, pp. 27, Paris. [Published in (Rodrigues 1816*f*).]

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(1816*d*) De la manière d'employer le principe de la moindre action, pour obtenir les équations du mouvement rapportées aux variables indépendantes. *Correspondance sur l'École Impériale Polytechnique*, **3**, no. 2, May 1815, 159–162. [See (Hachette 1808).]

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CHAPTER 8

Olinde Rodrigues's Paper of 1840 on a Group of Transformations

Jeremy Gray

Introduction

In this chapter we discuss a paper by Olinde Rodrigues which became almost completely forgotten and which is perhaps the first treatment of the subject of groups of motions. This is his '*Des lois géométriques qui régissent les déplacements d'un systéme solide dans l'espace, et de la variation des coordonnées provenant de ces déplacements considérés indépendamment des causes qui peuvent les produire*', which was published in 1840 in Liouville's *Journal de Mathématiques*, Volume 5, pages 380–440. We shall proceed by discussing (1) the contents, (2) the context, and (3) the significance of the paper.[1] The circumstances of its publication in relation to Rodrigues's life are discussed in Chapter 2.

The contents of Rodrigues's paper

As the title of his paper indicates, Rodrigues studied the motions (déplacements) of a rigid body (système solide) in three dimensional space independently of any dynamical considerations. He began by giving a complete description of motions in synthetic terms, establishing successively that a body is fixed in space once three noncollinear points have been determined; that if two points are fixed the motion is a rotation about an axis through those points; that translations when composed give a translation which is independent of the order of composition and can be found by the parallelogram law for translations ('loi du polygone des translations' (Rodrigues 1840, p. 383)); and that a translation is equal to an infinitesimal rotation about an axis perpendicular to the direction of the translation but situated at an infinite distance.[2] This last observation allowed him to consider translations as a special class of infinitesimal rotations.

To describe a general motion, Rodrigues noticed, as Euler ((Euler 1758, §2), (Euler 1765, §690)) had before him, that it can always be factored as a rotation followed by a translation. Indeed, let P be any point on the solid and P' its image under the motion. The translation taking P to P' puts the solid in what Rodrigues called an intermediate position. The rotation about P which moves the solid from its initial position to its intermediate position, followed by the translation from

[1] This is a modified version of a paper published in *Archive for History of Exact Sciences*, **21** (1980), 375-385. Reprinted with kind permission of Springer Science and Business Media. Material covered elsewhere in this book has been removed, and the references have been updated.

[2] Rodrigues described the rotation as 'of an amplitude infinitesimally small around a fixed axis infinitely distant and normal to the direction of this translation' (Rodrigues 1840, p. 381).

P to P', is the sought-for factorization of the original motion. This factorization is not unique, for the position of the axis of rotation may be altered if a different point P is chosen. However, he showed that this only varies the position of the axis, not its direction or amount (Rodrigues 1840, p. 385). This result follows from the fact that a rotation about one axis is equal to a rotation through an equal amount about a parallel axis, followed by the translation which sends a point on the second axis to its image under the first rotation, and this observation seems to be due to Rodrigues. The nature of the factorization permitted him to define the absolute translation, t, of any motion (Rodrigues 1840, p. 386) as the value of the projection of PP' onto the axis of the rotation part of the motion, where by the projection of PP' he meant the feet of the perpendiculars from P and P' to the axis. This value must be a constant independent of P, and it vanishes precisely for those motions which are composed of a rotation and a translation perpendicular to the axis of rotation, which are, of course, pure rotations about a suitably chosen parallel axis. The absolute translation is the measure of the translation part of screwlike motions, those composed of a rotation and a translation parallel to the axis of rotation, and the position of the axis of a general motion can always be chosen so that it is the resultant (résultant) of a rotation and a translation along the axis (Rodrigues 1840, *Théorème fondamental*, p. 385).

Rodrigues showed that any motion can be factorized in infinitely many ways as a product of two rotations about fixed intersecting (convergents) axes and then concentrated on theorems about the composition of motions. He had already considered the composition of two translations and of a translation and a rotation. He had finally to consider the composition of two rotations about axes which might be coincident or parallel or nonintersecting. The case of nonintersecting axes reduces to that of intersecting axes once the composition of a rotation and a translation is understood, for the axes can be made to coincide by means of a translation. In the case of parallel axes the combined motion is a rotation about a third, parallel, axis, the position of which depends on the order of composition. The combined effect of two rotations about coincident axes, he showed, was again a rotation, the size of which did not depend on the order of composition but the axis of which did depend essentially upon the order. However, he remarked, the combination of two infinitesimal rotations is independent of their order, for they combine like infinitesimal translations which can be computed by resolving them as vectors along coordinate axes.

The synthetic description of motions in space being now concluded, Rodrigues then reexpressed all his results in analytic terms. Of most interest to us is his derivation of the equations corresponding to a rotation expressed in terms of four parameters and his discovery of the rule for combining these parameters when two rotations are performed successively.

To describe a rotation, Rodrigues took a rectangular coordinate system Ox, Oy, Oz with origin on the axis of rotation, and he specified the axis in terms of its direction cosines $\cos g, \cos h, \cos l$ with respect to the coordinate axes; he called the amplitude of the rotation θ .

The formulae for a general motion involving a rotation and a translation were not expressed by Rodrigues in a particularly simple form (Rodrigues 1840, p. 399), but for a pure rotation they reduce to a more elegant set as follows. Suppose the point (x, y, z) goes to $(x + \Delta' x, y + \Delta' y, z + \Delta' z)$. Let (X', Y', Z') be the midpoint

of the motion, i.e., $(X', Y', Z') = (x+\Delta' x/2, y+\Delta' y/2, z+\Delta' z/2)$; then he showed (Rodrigues 1840, p. 407, correcting misprints):

$$\Delta' x = 2\tan\left(\frac{\theta}{2}\right)(Y'\cos l - Z'\cos h),$$
$$\Delta' y = 2\tan\left(\frac{\theta}{2}\right)(Z'\cos g - X'\cos l),$$
$$\Delta' z = 2\tan\left(\frac{\theta}{2}\right)(X'\cos h - Y'\cos g)$$

from which the nowadays more familiar expressions relating $(x+\Delta' x, y+\Delta' y, z+\Delta' z)$ to (x, y, z) may easily be obtained, although Rodrigues did not do so.

His next task was to follow the first rotation with a second, parameterized by θ', g', h', l', and to express the parameters Θ, G, H, and L of the combined rotation in terms of the eight parameters $\theta, g, \ldots, l'$. In the simplest case the two axes of rotation meet at the coordinate origin. He then found by elementary algebra that if v is the angle between the two axes, so

$$\cos v = \cos g\cos g' + \cos h\cos h' + \cos l\cos l',$$

then

$$\tan\left(\frac{\Theta}{2}\right)\cos G$$
$$= \frac{\tan(\frac{\theta}{2})\cos g + \tan(\frac{\theta'}{2})\cos g' + \tan(\frac{\theta}{2})\tan(\frac{\theta'}{2})(\cos h\cos l' - \cos l\cos h')}{1 - \tan(\frac{\theta}{2})\tan(\frac{\theta'}{2})\cos v}$$

with similar formulae for $\tan(\frac{\Theta}{2})\cos H$ and $\tan(\frac{\Theta}{2})\cos L$, and for the amplitude he found $\cos\frac{\Theta}{2} = \cos\frac{\theta}{2}\cos\frac{\theta'}{2} - \sin\frac{\theta}{2}\sin\frac{\theta'}{2}\cos\nu$.

For the inclination of the axis of the combined rotation he found

$$\sin\frac{\Theta}{2}\cos G = \sin\frac{\theta}{2}\cos\frac{\theta'}{2}\cos g + \sin\frac{\theta'}{2}\cos\frac{\theta}{2}\cos g'$$
$$+ \sin\frac{\theta}{2}\sin\frac{\theta}{2}(\cos h\cos l' - \cos l\cos h')$$

with similar formulae for $\sin\frac{\Theta}{2}\cos H$ and $\sin\frac{\Theta}{2}\cos L$.

He remarked that Θ is clearly independent of the order of combination of the two rotations. However, if one sets $\cos l = 0 = \cos l', \cos h = 0, \cos g = 1, \cos g' = \cos v, \cos h' = \sin v$, as one may without loss of generality, then $\sin(\frac{\Theta}{2})\cos L$ reduces to $\sin(\frac{\theta}{2})\sin(\frac{\theta'}{2})\sin v$ which changes sign when the order of combination is reversed, whereas $\sin(\frac{\Theta}{2})\cos G$ and $\sin(\frac{\Theta}{2})\cos H$ are unaltered. In fact, this is only correct as far as the expression for $\sin(\frac{\Theta}{2})\cos G$ is concerned, but that did not affect the validity of his conclusion, which was that the order of combining two rotations affects the axis of the final rotation but not its amplitude, as he had earlier shown synthetically.

Rodrigues gave analytic formulae for all the possible motions and their combinations, finite and infinitesimal, rotations about various axes, and translations. The infinitesimal rotation is of some interest. He found that it was described by

the equations (Rodrigues 1840, p. 402)

$$\delta x = \alpha + \theta y \cos l - \theta z \cos h,$$
$$\delta y = \beta + \theta z \cos g - \theta x \cos l,$$
$$\delta z = \gamma + \theta x \cos h - \theta y \cos g$$

about a central axis defined by the equations

$$\frac{\theta x + \gamma \cos h - \beta \cos l}{\cos g} = \frac{\theta y + \alpha \cos l - \gamma \cos g}{\cos h} = \frac{\theta z + \beta \cos g - \alpha \cos h}{\cos l}.$$

He then showed that two infinitesimal rotations about the same axis combined additively (Rodrigues 1840, p. 414). On pp. 421–430 he rederived his equations for a general motion purely algebraically, and then on pp. 430–432 he made the significant observation that the infinitesimal motions can be integrated to give the finite motions.[3] Specifically, letting the finite change in x be Δx and the infinitesimal change be δx, etc., he deduced from $\Delta(dx^2 + dy^2 + dz^2) = 0$, which expresses the rigidity of the body, that the following six equations for (x, y, z) and $(X, Y, Z) = (x + \frac{1}{2}\Delta x, y + \frac{1}{2}\Delta y, z + \frac{1}{2}\Delta z)$ hold (Rodrigues 1840 p. 431, correcting misprints):

$$\frac{d}{dX}\Delta x = 0 = \frac{d}{dY}\Delta y = \frac{d}{dZ}\Delta z,$$
$$\frac{d\Delta x}{dY} + \frac{d\Delta y}{dX} = 0 = \frac{d\Delta x}{dZ} + \frac{d\Delta z}{dX} = \frac{d\Delta y}{dZ} + \frac{d\Delta z}{dY}.$$

These integrate easily to give $\Delta x = A + pY - nZ, \Delta y = B + mZ - pX, \Delta z = C + nX - mY$ from which his earlier formulae for a general motion may be obtained. The paper concludes with a brief application of the infinitesimal analysis to a problem in statics. He establishes what he calls the equation of virtual velocities ('équation des vitesses virtuelles' (Rodrigues 1840, pp. 438–439) which says: 'the forces acting on a solid system being in equilibrium, if the system is moved infinitely little from its actual position by whatever cause, the sum of the forces multiplied by the infinitely small distances the points of the system run through in the direction of these forces must be zero, and conversely, and this is the statement of the principle of virtual velocities.'

The context of the paper

To appreciate the significance of Rodrigues's paper, it is necessary to look briefly at what work on the subject of rigid body motions had already been done. That subject forms part of dynamics and statics, an immense field of mathematical research, but it is precisely mechanics from which Rodrigues was seeking to abstract his discoveries. By studying the laws governing the displacements of a rigid body independently of the causes which can produce them, he made an abrupt break with the tradition of Euler. In Euler's work it is the forces upon a rigid body and their effects which are discussed, and the geometry of motions is developed only incidentally. Thus Euler did establish that every motion can be expressed as a combination of a rotation and a translation (Euler 1758, 1765) and that every rotation of a rigid body is about an axis which is instantaneously fixed (Euler 1758, §3). But his chief concern was with the effect of impressing given forces upon

[3]Formulae for infinitesimal motion were given by Lagrange in his *Mécanique Analitique* of 1788 in the section 'La Statique', pp. 55–58, where the composition was stated to be additive.

a body and in finding what forces will produce a desired effect. The techniques Euler employed are those of analysis. In Rodrigues's paper all that is considered are the abstract motions. So it seems that Euler is the first to give an explicit notation for describing a rotation, the so-called 'Euler' angles, but that Rodrigues is the first to give a formula for the combination of two rotations as expressed in terms of four parameters, as Klein remarked (Klein 1884, p. 38). However, Jacobi observed (Jacobi 1881) that Euler had considered the combination of orthogonal matrices[4] in his number-theoretic research (Euler 1770), and there he composed the general orthogonal rotation in three, four, or five dimensions as a product of rotations in successive planes, as he also did in his mechanics (Euler 1775*a*) in three dimensions. Formulae for finite motions were given by Monge (Monge 1784) and in a simplified form by Chasles (Chasles 1837*b*, p. 678), neither of whom sought the rule for combining motions. Chasles' geometrical derivation (Chasles 1830) of Euler's results in $\mathbb{R}^3$ influenced Rodrigues's methods, and Rodrigues attributed some of the theorems about a single motion to Chasles (Rodrigues 1840, p. 386).

The aim of Rodrigues's paper, though, was not to calculate the composition of two rotations, a rather easy and uninteresting task, but to draw attention to the motions of a rigid body as an important object of study. Euler gave a set of four parameters which describe a rotation: p, q, r, and $\cos\phi$ which satisfy the equation $p^2 + q^2 + r^2 = 1$ for a fixed but arbitrary ϕ (Euler 1775*b*, p. 124). Rodrigues supplied a similar set of parameters g, h, l, and θ and gives the formulae for the composition of two rotations in terms of these parameters. As Klein (op. cit.) remarked, this is equivalent to Hamilton's system of quaternions. Klein incorrectly implied that geometric research governed Hamilton's research prior to his discovery of quaternions. In fact Hamilton came to his discovery of quaternions at the end of a long series of algebraic research, and he said that he only discovered the connection between unit quaternions and rotations of the unit sphere the next day (Hamilton 1843*a*, 1843*b*). Klein here was guided by Cayley's remarks (Cayley 1845) in which Cayley observed that if q denotes the quaternion $\alpha + \lambda i + \mu j + \nu k$ and $xi + yj + zk$ is a unit quaternion (and so lies on the unit sphere in $\mathbb{R}^3$), then a rotation of the sphere is given by $(ix + jy + kz) \to q^{-1}(ix + jy + kz)q$. This rotation[5] is through θ about an axis through the centre of the sphere which has direction cosines $\cos f, \cos g, \cos h$ given by $\lambda = \tan(\frac{1}{2}\theta) - \cos f, \mu = \tan(\frac{1}{2}\theta) - \cos g, \nu = \tan(\frac{1}{2}\theta) - \cos h$. This is the first published account of the connection between quaternions and rotations, but it is not the first time it was discovered (cf. (Gauss 1819)).[6] Cayley conceded priority in a footnote to the paper when it was republished in the first volume of his *Collected Mathematical Papers*. Cayley had earlier (Cayley 1843) given a somewhat unoriginal rendition of Rodrigues's paper, as Hawkins (Hawkins 1977) has observed, and in (Cayley 1848) he gave a quaternionic formulation of

[4]Euler was interested in arrays of coefficients $X = Ax + By + Cz$, $Y = Dx + Ey + Fz$, $Z = Gx + Hy + Jz$ which satisfied the conditions $A^2 + B^2 + C^2 = 1$, etc., and $AD + BE + CF = 0$, etc. He presented the array sometimes within the context of simultaneous linear equations where the quantities $X^2 + Y^2 + Z^2$ and $x^2 + y^2 + z^2$ are to be equal, sometimes shorn of the variables as *novem numeros ita in quadratum disponendos* (Euler 1770 p. 288). Similarly for the 4-by-4 case (Euler 1770 p. 313).

[5]The occurrence of half-angles gives a twofold covering of the rotation group; see (Chevalley 1946).

[6]It seems that Gauss invented his quaternions to simplify the composition of rotations in $\mathbb{R}^3$, but he never published his discovery. They are Hamilton's complex conjugates.

other results by Rodrigues. In these two papers he considered their implications for mechanics, thus directing attention back to Euler's concerns. It is striking that he did not feel inclined to appreciate the novelty of the abstract objects here being introduced into mathematics; neither here nor later, when he was studying groups and matrices, did he seek to recall Rodrigues's work. It is perhaps for this reason that Rodrigues is only remembered, if at all, for his formula for multiplying quadruples (θ, g, h, l). When, for instance, Jordan (Jordan 1867, 1868) gave a group-theoretic interpretation of Bravais's (Bravais 1849) work on crystal lattices in terms of discrete subgroups of the group of rigid body motions in Euclidean space, Rodrigues's name is not mentioned. Jordan merely remarked 'One knows that every displacement of a solid body in space is a helicoidal movement and given two such movements one easily constructs the resulting movement which will also be helicoidal[7]

Bravais did not consider motions in space and did not mention Rodrigues. It seems that Klein's confused and possibly second hand description of Rodrigues's work is the only major nineteenth-century source to connect the paper (Rodrigues 1840) with the early history of transformation groups.[8] In any event, Rodrigues's paper is not mentioned in Wussing's thorough survey of the topic (Wussing 1969). Coolidge's earliest reference (Coolidge 1940) to motions as an object of study is (Poinsot 1851). This long paper is, however, chiefly devoted to giving a vivid description of how a solid body moves in space in terms of its concurrent rotations or screw motions about three axes in elliptic cones (Poinsot 1851, Part 3, §37). Most likely Jordan had read this paper. In keeping with the dynamical aims of the paper, forces are resolved into couples producing the screw motions, and Poinsot referred to Euler but not to Rodrigues even though his paper was also published in Liouville's *Journal*.

In other geometries studied at this time only an implicit reference to transformations is found; figures are studied which are unaltered by all projective transformations in (Poncelet 1822) and (Möbius 1827). No reference to combining transformations was made by Chasles in (Chasles 1837*a*), and no work on non-Euclidean geometry discussed the combination of motions for at least another generation. This is in keeping with the preference for invariants over symmetries that marks much of algebra and geometry before about 1870.

Rodrigues has himself become a somewhat shadowy figure, rating no entry in Poggendorff's bibliographies, (Sarton 1957), (May 1973), or the *Dictionary of Scientific Biography*. Kline (Kline 1972, pp. 531, 883) records that he was born in 1794, is the author of Rodrigues's formula for the Legendre polynomials and of various research into differential geometry, notably curvature of surfaces (see also (Reich 1973)), and that he died in 1851. The *Royal Society Catalogue* (1871) lists several papers by him in the *Correspondance de l'École Polytechnique* (1814–1816) and Liouville's *Journal* (1838–1843) but only one on the theme of groups of motions. Bourbaki comments dryly but accurately in a footnote (Bourbaki 1969, p. 164f2)

[7] 'It is known that any displacement of a solid body in space is a helicoidal motion and, two such motions being given, one can easily construct the resultant motion which will itself be helicoidal' (Jordan, 1868, p. 229).

[8] I do not know if Klein had read Rodrigues's paper or merely took his information from Cayley. The latter hypothesis is supported by the fact that Klein always associated Rodrigues with combining quadruples and compared his method with those involving quaternions, e.g., (Klein 1926, Vol. 1, p. 186), but he never mentioned Rodrigues in connection with groups of motions.

that Rodrigues discovered his formula shortly before falling into the oblivion of the nineteenth century.

The significance of the paper

Rodrigues's paper certainly deserves, because its thoroughness, to be considered as a paper in transformation groups, specifically on the group of isometries of Euclidean three-space. It is not, primarily, concerned with dynamics or statics, with the causes and manner of motion of rigid bodies. The laws of motion are studied abstractly, with a view to establishing the rules for combining motions. This point of view is precisely that taken later by Klein in the *Erlanger Program* (Klein 1872). There Klein defined a transformation group as a set of motions closed under the operation of taking products; only later did he add the condition that inverses must exist, even then taking the existence of an identity element for granted. Rodrigues gave a complete description of how to combine the two basic motions, rotations and translations. In modern terms, he exhibited the group of proper Euclidean isometries of three-space as a semidirect product of the proper orthogonal group with the group of translations. He proved that the group of translations is commutative, and he treated its elements as vectors, resolving them along the coordinate axes. On the other hand, he was clearly concerned to present the orthogonal group, as noncommutative. Three points are particularly noteworthy: the stress he places on the noncommutativity of this group; the contrast with the infinitesimal rotations, which do commute; and the realization that the infinitesimal motions generate the finite ones.

Rodrigues's concern that the order of composition matters is patent. It is not a mere cautionary aside nor an almost unremarked consequence of the formulae, but it is a matter for him of the greatest significance, reflecting his abstract approach to the subject matter. He was explicitly drawing attention to a novel property of the abstract objects (motions) that he was introducing into mathematics. He stressed in several places that the order of composition matters, before giving the matrix formulae for combining rotations, where it might have been allowed to pass unheralded. The attention he paid to the rules of combination is therefore most interesting.

There is a sense in which it had been well known for many years that matrices do not commute: the programme of simplifying systems of linear equations by change of basis does not make sense, for instance, if all matrices commute. But this awareness was covert, so to speak, and is not the same as an explicit consideration of the rules for combining matrices. It seems that the first person to draw attention to the noncommutative nature of matrix multiplication was Eisenstein in 1844 (quoted in (Hawkins 1977, p. 85)). As Hawkins (Hawkins 1977) has shown, Eisenstein also had the idea of regarding a matrix as a single object and supplied single letter notation for it, which Rodrigues did not, but by preferring to reason geometrically rather than analytically, Rodrigues clearly conceived of each isometry as a single thing, an array.

It is interesting to note that other noncommutative systems were introduced successfully into mathematics at about this time, although earlier attempts had met with less success. Galois' profound remarks on permutation groups (Galois 1832) elicited no response at all for many years, as is well known. It is hard to determine the impact of Abel's remarks, e.g., (Abel 1829), on the connection between

the solvability of certain polynomial equations and the commutativity of the group of permutations of their roots, but it would seem that Cauchy's study (Cauchy 1845) is the first time Cauchy, at least, took the conspicuously noncommutative behaviour of permutations seriously; his earlier studies, done around 1815, contain no explicit reference to it. There was a small French tradition of studying symbolic methods in analysis, and it was there that the word commutative was first introduced into mathematics in 1815. Servois said of two functions F and f that they were 'commutatives entre elles' if $F(f(x)) = f(F(x))$ (Servois 1815), but this formal analysis was disapproved of in France for its lack of rigour, as Koppelmann has discussed (Koppelmann 1971). The tradition did much better when transplanted to British soil, and in 1837 Murphy began the study of noncommutative operators in the context of differential equations. This algebraic approach to analysis was greatly refined by Boole (Boole 1844), and its general acceptance into mathematics dates from then.

Grassmann's largely unread *Ausdehnungslehre* appeared in 1844, but undoubtedly the paper most immediately related to (Rodrigues 1840) is Hamilton's 1843 announcement of the discovery of quaternions.[9] As has been remarked, Hamilton came to his discovery algebraically and only put the geometrical considerations first in his later writings. Hamilton's quaternions are numberlike, they are generalizations of complex numbers, and their noncommutativity resounds throughout the subsequent study of algebra. Rodrigues's motions are geometric, but both point toward new, abstract, totalities worthy of the mathematicians' attention. (Further discussion of the implications of Rodrigues's paper on the theory of quaternions may be found in Chapters 2 and 9.)

Indeed, the sudden emergence of noncommutative systems of abstract entities into mathematics in the early 1840s is of considerable historical interest. At the risk of banality, it should be pointed out that all these systems arose from real problems in mathematics. Matrices, at least as arrays of coefficients, abounded in contemporary problems. Permutations derive from the study of polynomial equations, with the unsolvability of the quintic as a central topic. Quaternions are a natural extension of the then best description of real and complex numbers. In each case, except the last, some attention is switched from the study of, or search for, a particular object of a given kind to the study of all objects of that kind; Hamilton's approach was necessarily directed towards the entire algebra. So one could look for the origins of the abstractness of much of modern mathematics in the work of this period. It could also be said that this period saw the emergence of the mathematical profession as a sizable body of people, with substantial schools developing in Britain (Boole was 25 in 1840; Cayley, 19; Hamilton, 35; Sylvester, 26), Germany (preeminently Dirichlet, then 35; Eisenstein, 17; Jacobi, 36; Kummer, 30; and Weierstrass, 25), and Russia, as well as a powerful new generation in France (Hermite, 18; Liouville, 31) to name only a few. It might be that the growth of the profession itself spurred mathematicians to seek abstractions from within mathematics as well as from neighbouring disciplines. Klein in his *Entwicklung* (Klein 1926) suggested that whereas the great mathematical physicists of 1800 belong equally to mathematics and physics, by 1850 a separation of the subjects

[9]There is no reason to suppose Hamilton had read Rodrigues's paper.

is beginning with the appearance of physicists (Maxwell, Helmholtz) whom mathematicians have not been able to claim as their own. Perhaps the gulf was then beginning to inhibit the easy movement between disciplines.

Whatever the validity of these highly tentative speculations, they will not be pursued further here. Rodrigues's paper itself did not draw its audience towards abstraction. Jacobi and Klein saw in it a pleasing calculus, and Cayley redirected it towards problems in mechanics; between them they returned its topic to various aspects of the extensive work of Euler from which the nineteenth century grew so energetically. In so doing they missed its most important idea, which was only to be rediscovered a generation later by Jordan, Klein, and Lie: that of the transformation group.

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