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PREFACE

This book arose out of a course entitled

Mathematics in ancient and modern times:

The birth of modern algebra

which I gave to a small group of freshmen students in UCLA in the Fall of 1993. The students had only high school mathematics as their background and so I did not use calculus. However I assisted them so that they selected individual projects to work on as their assignment for fulfilling the requirements of the course. Each project was designed to have two parts: a historical and a mathematical one. The projects had wide variety which surprised me, but which increased my pleasure as the course progressed. This was the first time that I had taught a small group of freshmen in a format different from the usual cook-book type in more than thirty years of teaching in UCLA, and the experience was memorable.

My inspiration for designing and teaching such a course came after reading the beautiful book *Tales of physicists and mathematicians* by my friend Simon Gindikin, especially his account of the life and work of Cardano and its influence on the rise of modern algebra. However in the course I did not confine myself to the work of Cardano but explored a wider theme, namely the growth of algebra from its ancient origins to its current state that allows us to have a marvellous understanding of the whole gamut of physical sciences: from the abstract reaches of number theory to the wonders of quantum physics.

In 1996 Rajendra Bhatia of the Indian Statistical Institute suggested the possibility of making a book out of these notes. I then revised the notes adding exercises and additional material—partly historical and partly dealing with more advanced topics—in the form of notes and exercises at the end of the individual chapters. These may also be used as starting points for projects for the more ambitious student. The result is what I hope is an interesting way to present the basics of modern algebra at a very elementary level. The approach that I have adopted here (as in the course) is what Shafarevich calls the biogenetic one; the student is exposed to the historical and evolutionary development of the subject as an essential part of the course, and the material is presented in a way that allows him or her to see clearly how ideas have emerged in the attempt to solve specific questions.

I am not sure how successful my attempt has been. However it was a great experience for me, not only at the time the course was given, but also later when the book was written. I would like to record my gratefulness to the Honors Collegium of UCLA which encouraged me to give the course, to the small group of young men and women who took the original course, and to R. Bhatia who encouraged me

to enlarge the notes into its present form. Finally, I would like to thank Wissam Barakat for his great technical help in the preparation of the final version of the manuscript.

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