Preface

Migratory animals such as whales, caribou, sea turtles, butterflies, and birds seem to have built-in maps that enable them to travel between precise locations without any apparent external assistance. Humans have also migrated considerably, from our origins in Africa to Asia, Oceania, Europe, and the Americas, and have otherwise travelled extensively for thousands of years. Evidence of maps used to describe our surroundings, to interpret the natural environment, to orient ourselves, and to aid in navigation dates back as far as our knowledge of human civilization. Although we can't say for certain when the first maps were made or for what specific purpose, it seems likely that some of the earliest maps were used to tell where a given something or somewhere was located – a productive fishing spot, fresh water, or a dangerous place to be avoided.

Today we encounter maps on a daily basis. Maps from the simple and sublime to the complex and confusing. From the symbols on restroom doors to street maps of the towns and cities where we live, from shopping mall directories to detailed geologic maps depicting the underground structure of the earth, from traffic signs to world maps accompanying newspaper articles about global events – all are maps. They guide and direct our daily routine activity; they help us to make difficult decisions. They can be used to present information, to formulate policies, or to explore. Whatever its purpose, every map is a tool, a product of human effort and creativity, that represents some aspects of our world or universe. Every map is a representation, an approximation, a reduction of something that is larger or more complex.

In order to make its point, every map actually distorts the truth, distorts reality, by employing some type of code to formulate and express its message. For many maps, one ingredient of this code is a mathematical transformation of the map's subject. This is most readily apparent, perhaps, in the sorts of maps shown in a typical atlas, but it is true of many other maps as well. In this book, we will focus on world maps, maps that portray relatively large portions of the earth's surface. We will study the different attributes that maps can have and determine mathematically how to construct maps that have the features we want. We will see how maps are used to communicate and how the distortions that arise in making maps of the world affect our ability to communicate. We will use maps to explore and to navigate across the seas and through the air. Along the way, we will learn of the longstanding collaboration and interrelationship between geography and mathematics. We will travel to every continent, visit civilizations both ancient and contemporary, and meet a few of the many outstanding individuals who have influenced the way we look at and map our world.

This book grew out of an interdisciplinary undergraduate course in mathematics and cartography designed by the author and the late Dr. Elaine F. Bosowski at

Villanova University. The course was powered by our belief that, by exploring the mathematical ideas involved in creating and analyzing maps, students would see how mathematics could help them to understand and explain their world. At the same time, by actually creating maps of their own, students could develop the skills needed to accurately evaluate the quality of graphic images they encounter on a routine basis. In other words, the interface between mathematics and cartography seemed to us to be an exciting and accessible area of study for college students – well, we were excited by it anyway! We called our course *Cartographiometry*, developed it with support from the National Science Foundation's Mathematics Across The Curriculum program (grant # NSF-DUE-95-52464), and offered it to students at Villanova University during the 1996-97 academic year. In the class, the students explored, as we will do in this book, the shape of the earth, the determination of latitude and longitude, elementary spherical geometry, the uses and computation of scale factors, the design of optimal routes for air or sea navigation, Gaussian curvature, the reasons why we can't make a perfect flat map of the earth, how to evaluate from a critical and analytical viewpoint maps that we encounter every day, and how to design atlas maps using both hand-drawn techniques and computer graphics software. Since Dr. Bosowski's untimely death in 1998, variants of the *Cartographiometry* course have been taught in a distance-learning format and as a senior mathematics seminar.

In this book, the intent is not to give a historical overview of humankind's attempt to map our world, but to investigate in some detail the mathematics involved in doing so. Necessarily, the discussion will become somewhat technical at times as we work through the analysis and equations needed for one map or another. This may seem a bit tedious in places, but there is no escaping it other than to accept someone else's word on how to design a map. If we want to understand this process and be able to construct and analyze maps for ourselves, we must pursue the mathematical details.

Despite its more or less obvious presence, the mathematics behind various maps is rarely discussed in any undergraduate mathematics or geography course, except, possibly, as an interesting side topic in a first course in differential geometry taken by advanced mathematics and physics students (cf. [McCleary, 1994]). Some books on map projections written by cartographers (e.g., [Pearson, 1990], [Bugayevskiy and Snyder, 1995]) take this approach as well, though few undergraduate students in geography have the mathematical prerequisites to use them. Other books, such as the classic text of Deetz and Adams, present tables of calculations indicating how to construct various maps while essentially suppressing the actual mathematics. Most general cartography textbooks (see [Dent, 1996] and [Robinson, 1995]) include a brief overview of map projections and their properties with no attempt at mathematical analysis. Of course, as both the earth and the flat piece of paper onto which it is to be mapped are two-dimensional surfaces, a differential geometry approach, together with the background in multivariable calculus it presupposes, might seem to be necessary for understanding maps. In fact, this is not the case. As we shall see here, an important key to understanding certain properties of projections, such as the distortion of areas or angles, is an analysis of a map's scale factors. It turns out that calculating scale factors involves only the comparison of linear distances, so the basic elements of one-variable calculus usually will be adequate to our task.

The principal mathematical tools we will use are basic calculus – derivatives and integrals – and trigonometry. Many of the maps we will study were originally developed centuries or even millenia ago, and there is a significant gap in the levels of familiarity with trigonometry between mathematicians of the nineteenth century and earlier and we mathematical thinkers of today with our handy computational machines. So we should not be surprised if we must dust off some half-forgotten trigonometric relationships in the course of our investigations. A few sections of the book draw on the concept of vectors in two- or three-dimensional space, and one chapter (on obliquely centered maps) employs three-by-three matrices and a bit of linear algebra as central tools. You will miss some interesting ideas, but still get the main gist, if you skip those sections.

Many of the world maps that you see throughout the text were generated using the *Maptools package* created during the spring of 2001 by Vincent Costanzo, who was a student at Villanova University at the time. The package, which uses a database of coastal points developed by the United States Geological Survey, works within the computer algebra system Maple and is available at the Maple applications center website.

I owe an immeasurable debt to Elaine Bosowski, late of the Villanova University Geography Department. This book would not have been written without the collaboration we formed in the years before her death. I would also like to thank the students who took the *Cartographiometry* course in any of its various forms; the National Science Foundation, and especially Lee Zia, for supporting the *Cartographiometry* project; all those colleagues who responded with ideas and encouragement to various talks I have given on this subject; Gary Michalek for reading the whole manuscript carefully; Bob Jantzen for generously sharing his knowledge of LaTeX and Maple; and Ed Dunne, at the American Mathematical Society, for his invaluable support and advice. I take full responsibility for any shortcomings and mistakes that remain and welcome comments from readers.

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