
Preface

In February of 2007, I converted my “What’s new” web page of research updates into a blog at `terrytao.wordpress.com`. This blog has since grown and evolved to cover a wide variety of mathematical topics, ranging from my own research updates, to lectures and guest posts by other mathematicians, to open problems, to class lecture notes, to expository articles at both basic and advanced levels. In 2010, I also started writing shorter mathematical articles on my Google Buzz feed at

`profiles.google.com/114134834346472219368/buzz` .

This book collects some selected articles from both my blog and my Buzz feed from 2010, continuing a series of previous books [Ta2008], [Ta2009], [Ta2009b], [Ta2010], [Ta2010b], [Ta2011], [Ta2011b], [Ta2011c] based on the blog.

The articles here are only loosely connected to each other, although many of them share common themes (such as the titular use of *compactness and contradiction* to connect finitary and infinitary mathematics to each other). I have grouped them loosely by the general area of mathematics they pertain to, although the dividing lines between these areas is somewhat blurry, and some articles arguably span more than one category. Each chapter is roughly organised in increasing order of length and complexity (in particular, the first half of each chapter is mostly devoted to the shorter articles from my Buzz feed, with the second half comprising the longer articles from my blog).

A remark on notation

For reasons of space, we will not be able to define every single mathematical term that we use in this book. If a term is italicised for reasons other than

emphasis or for definition, then it denotes a standard mathematical object, result, or concept, which can be easily looked up in any number of references. (In the blog version of the book, many of these terms were linked to their Wikipedia pages, or other on-line reference pages.)

I will, however, mention a few notational conventions that I will use throughout. The cardinality of a finite set E will be denoted $|E|$. We will use the asymptotic notation $X = O(Y)$, $X \ll Y$, or $Y \gg X$ to denote the estimate $|X| \leq CY$ for some absolute constant $C > 0$. In some cases we will need this constant C to depend on a parameter (e.g., d), in which case we shall indicate this dependence by subscripts, e.g., $X = O_d(Y)$ or $X \ll_d Y$. We also sometimes use $X \sim Y$ as a synonym for $X \ll Y \ll X$.

In many situations there will be a large parameter n that goes off to infinity. When that occurs, we also use the notation $o_{n \rightarrow \infty}(X)$ or simply $o(X)$ to denote any quantity bounded in magnitude by $c(n)X$, where $c(n)$ is a function depending only on n that goes to zero as n goes to infinity. If we need $c(n)$ to depend on another parameter, e.g., d , we indicate this by further subscripts, e.g., $o_{n \rightarrow \infty; d}(X)$.

Asymptotic notation is discussed further in Section 3.5.

We will occasionally use the averaging notation

$$\mathbf{E}_{x \in X} f(x) := \frac{1}{|X|} \sum_{x \in X} f(x)$$

to denote the average value of a function $f : X \rightarrow \mathbf{C}$ on a non-empty finite set X .

If E is a subset of a domain X , we use $1_E : X \rightarrow \mathbf{R}$ to denote the *indicator function* of X , thus $1_E(x)$ equals 1 when $x \in E$ and 0 otherwise.

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terrytao.wordpress.com

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