Epigraph

> *Development of mathematics resembles a fast revolution of a wheel: sprinkles of water are flying in all directions. Fashion – it is the stream that leaves the main trajectory in the tangential direction. These streams of epigone works attract most attention, and they constitute the main mass, but they inevitably disappear after a while because they parted with the wheel. To remain on the wheel, one must apply the effort in the direction perpendicular to the main stream.*

—V. I. Arnold, translated from "Arnold in His Own Words," an interview with the mathematician originally published in *Kvant Magazine*, 1990 and republished in the *Notices of the American Mathematical Society*, 2012.

CHAPTER 7

About Vladimir Abramovich Rokhlin

V. I. Arnold

I first met Vladimir Abramovich Rokhlin at the seminar on ergodic theory at Moscow State University, and he would weekly commute from Kolomna where he was able to get his resident registration.[1] Coming to Moscow, he would stay at his friends' place and sleep on a folding bed. In the morning though he insisted on calling the folding bed differently than at night. "It's a folding bed in the morning, but an unfolding bed at night".

But spending every year as neighbors at the cottages at Nikolina Gora[2] for over 10 years had a much more significant impact on me. We would talk for hours about all sorts of things, usually strolling along the Moscow river bank, often accompanied by other Zarechie inhabitants – the Efimovs, the Shilovs, the Shura-Buras, the Jacobsons, the Kushnirenkos, the Pomanskis.[3] Sinai used to come to fill his water canister, because there was no running water in the nearby Novo-Daryino at that time.

According to Courant's definition, a mathematician should be considered young for as long as he is inclined to discuss math at the most inappropriate times. Moscow river bank would become a special kind of a remote office of the Mekh-Mat, filled with young mathematicians of all ages.

Speaking with Vladimir Abramovich[4] I always felt as if I were communicating with a supreme mind, aware of the most final and true answers to all questions. I felt that air of irrefutable assurance about him that I probably have never felt about anybody else I have known. Perhaps, only Dieudonné possessed a similar air of confidence in his judgments and opinions, but it was so obvious when he was wrong and he would become excessively agitated in the course of an argument (probably, due to inferiority complex of some sort, and that was completely foreign to Rokhlin's nature).

Vladimir Abramovich's dignified demeanor and physical appearance reminded me of Korney Ivanovich Chukovsky,[5] who used to visit us at Arbat (he went to

Originally published in Russian in the book "V. A. Rokhlin: Selected Works", A. Vershik (ed.), Second Edition, MCCME, Moscow, 2009.

[1]Kolomna is 112 km away from Moscow. As a former prisoner of Gulag, Rokhlin was not allowed to live closer than 100 km to Moscow.

[2]Nikolina Gora, Zarechie, Novo-Daryino are villages near Moscow where dachas (summer cottages) of many academics were located.

[3]Moscow mathematicians. Efimov served as the Dean of Mekh-Mat, the Faculty of Mechanics and Mathematics of the Moscow State University, in 1962–69.

[4]Rokhlin

[5]A famous children's poet, literary critic and essayist.

school with my grandmother's brother, B. S. Zhitkov,[6] and wrote very interesting things about him and my grandmother in his memoirs). Much later Vladimir Abramovich told me that this resemblance was no accident: Rokhlin and Chukovsky were closely related.

Although in a conversation, Rokhlin's air of irrefutable assurance could be irritating to others (not to me, I always listened to him with gratitude), this very aplomb added a unique charm to his lectures. And especially remarkable were V.A.'s lectures on topology, they entirely changed my views on what a good lecture is supposed to be.

One of the greatest science popularizers, Faraday, once said that the lectures that are really popular, are never instructive, and those, that are instructive, are never popular. Rokhlin's lectures managed to combine both virtues. Perhaps, the very reason why the book based on those lectures and written by D.B. Fuchs (a brilliant lecturer himself) is, in my opinion, no match for the lectures themselves is that while it is tempting to try and fit more material into a written text, this makes it difficult for a reader to follow the main idea.

Unlike Kolmogorov, who was hardly ever able to finish the phrase he started (let alone his proofs), Rokhlin was an articulate and effective speaker and never attempted to hide ideas behind calculations (which happened at times with his teacher, L. S. Pontryagin, whose brilliant lectures occasionally still lulled me to sleep). Rokhlin would just walk up to the blackboard, pick up a piece of chalk and start speaking. In about five minutes he would decide to use the board and write a letter on it (A, if he was talking about a ring, and M, if it was about a manifold, etc.). Then, a minute later he would erase what he had written, probably, so that the letter belonging to the part of the lecture that was over, would not divert the attention of the audience from the part that was to follow.

The audience (which was always too big to fit in the auditorium - undergraduates, graduate students, and professors, all attended his lectures) was listening in awe to the Great Master preaching. After all, the science Vladimir Abramovich was talking about had been all but banned on the Faculty (or at least had not been discussed in courses accessible to students) for over thirty years. The only topology allowed at Mekh-Mat was pathological (even now there still exists a chair of pathological topology here, perhaps the only one in the world).

To illustrate how detached we were from the rest of the world at the time, I should mention a curious incident. A remark found in the collection of translated papers "Fibre Bundles" that the idea of expressing a spectral sequence by rectangles consisting of groups, where the consecutive differentials act by generalized knight moves, is due to E. B. Dynkin (so that the corresponding figures in the notes to the translation are called Dynkin's diagrams). When I was in France in 1965, I asked Serre whether he knew of this improvement to the theory. Serre couldn't stop laughing. How else could one make calculations with spectral sequences? To be fair, in French publications there were no diagrams (indispensable to the reader) – probably to make the theory incomprehensible to the uninitiated (but more likely, due to typically careless French user-unfriendliness).

In his lectures, Rokhlin's used to reveal every such little secret one after another. Vladimir Abramovich was fully aware that no matter how much time one could save

[6] An author, mainly of children's books, based on his rich experience as a sailor, ship captain, and explorer.

using the deductive methods ("from general to specific"), the value of a lecture for a student consists of merely a number of well-explained and thoroughly understood examples. Vladimir Abramovich's attitude towards examples was that of a respect, similar to the one held by physicists of inductive school of thought (starting with Newton), and contrary to the opinion of most of contemporary mathematicians (Sullivan once told me that he tended to avoid dealing with particular examples at all costs – they were way too complicated).

What made Rokhlin's lectures stand out was the way he dealt with both examples and theories, based on his perfectly pragmatic approach. "It is the cycles that are the most useful," he used to say, "but the cycles are like underwear you are not supposed to display in public; what is left in articles is only homological classes."

Rokhlin's opinions of the mathematicians around us and their mathematical theories was as insightful (as I see it now), as undeservedly harsh it seemed to me then. However, I heard P. S. Alexandrov say many times that "the highest degree in this country is doctorate; everything higher than that has little to do with achievements in science". But in some cases (for example, towards Kolmogorov and Pontryagin) Rokhlin was much more tolerant. His definitive opinions and the way of expressing them was what distinguished Rokhlin from almost all the mathematicians I have known, and in that respect I consider myself to be his follower. Rokhlin himself thought that he inherited these qualities from his teacher Pontryagin, who was known to follow in this in Benvenuto Cellini's steps.

When in 1961 Milnor came in Leningrad to attend the All-Union Mathematical Congress and talked about non-standard smooth structures on spheres, the impact of the progress made in non-pathological topology on mathematics in whole became evident even for older generation of the mathematicians, who had been ignoring everything that happened in scientific research after 1935 (as a consequence of an almost total 15-years-long isolation of Russian Mathematical school from the Western one). At that point Rokhlin became practically the only Russian mathematician who was actively involved in global efforts in conquering this new and unknown mathematical continent. "Mathematics today," he would say to me, "is like an exclusive aristocratic club. On top of a tremendous initiation fee, a hefty annual fee is required from its members".

Influenced by Milnor's presentation, my scientific advisor Kolmogorov recommended that I, a graduate student at the time, would include Milnor's spheres in my dissertation plan. (He intended to find out from me what was going on in topology from then on). At his advice I started learning differential topology from Fuchs, Novikov and Rokhlin and in a year I was appointed to be a referee of Novikov at his dissertation presentation. It was impossible to find a referee among the older generation, since already the words "exact sequence" represented an insurmountable obstacle for our professors at that time.

The level of understanding of "contemporary mathematics" that existed at the time at Mekh-Mat can be illustrated by the following episode. When I started explaining to Kolmogorov "what was going on in topology", he responded that his four articles published in 1935 in *Comptes Rendus* (where he introduced cohomology, simultaneously with Alexander, but based on physics, the ideas of hydrodynamics and electromagnetic theory) went unnoticed and unappreciated by topologists. "As a matter of fact," he said, "I not only identified the cohomology *groups* (which were

understood by everybody) but I also introduced the *ring*. If topologists paid attention to the ring (even now) they would be able to obtain interesting new results."

While in my unsuccessful attempts I was naively determined to make Andrey Nikolaevich change his views on the world of mathematics, Vladimir Abramovich displayed an uncharacteristic tolerance in this case. "The assessment of cohomological multiplication given by Kolmogorov," he told me, "is twice as remarkable because it is an evidence of his understanding of the significance of his own achievements and at the same time it foresees the future role of cohomology operations in general!"

Vladimir Abramovich somehow managed to combine his dangerously uncompromising views with an unusually apparent self-confidence, which inadvertently earned him respect even among his enemies. Perhaps, the reason for it was that unlike the majority of the colleagues his age and older, Vladimir Abramovich managed to avoid the humiliating compromises with the authorities, which poisoned the lives of generations of our countrymen. It is clear, of course, that he was just lucky to survive and that he occupied an undeservedly low position in the official hierarchy of the Soviet mathematicians. (He was undoubtedly our best mathematician of his generation, who also greatly influenced the future development of mathematical science in Russia). But unlike Kolmogorov and Pontryagin for example, by and large he had no reason to reproach himself. (This was something that he had in common with I. G. Petrovsky, who also earned an involuntary respect even among his enemies).

Unlike Petrovsky, Rokhlin was at some point influenced by the brown plague of Nietzschean philosophy. Rokhlin's friends recall that before the war he had been quite impressed with Hitler, his determination and his success.

After having spent lengthy periods of time both in France and Germany I am less surprised now with the viewpoint of young Rokhlin. France all but followed in Nazi Germany's footsteps in 1933. The majority of French now believe that Hitler beat Russia in WWII, but that he was later, as I recently found out from my French colleagues, defeated by France under de Gaulle. In German schools even now children are taught the following: "The defeat in WWI left Germany in a very bad shape economically. Hitler saved Germany, but he committed some mistakes in his foreign policy, which in turn lead to the defeat of Germany in WWII." When I asked random people on the streets of Bonn, I found out that "Adolf wouldn't tolerate such a lack of order we have now" and that the years of his rule were the best years ever in Germany. I asked an old little lady in Dusseldorf, "What is the name of this street – Adolf Hitler or Count Adolf?"[7] She replied, "Adolf deserves it" (i.e., Adolf earned the title of a count).

There is nothing surprising in the popularity of pro-Hitler sentiments in contemporary Russia, including among mathematically educated folks! Rokhlin, as it seems, was cured of his pro-Hitler illusions in a Nazi camp (while a Stalin camp completed his education).

I can easily picture young Vladimir Abramovich (as per his fellow students' recollections) leaving Steklov Institute library building at night, stretching, and saying, "Today is not my day, proved only seven theorems."

[7]Count Adolf Street would be the equivalent of Yuri Dolgorukiy Street in Moscow (V. Arnold's footnote).

Our conversations with V.A. usually were more like his monologues. I gained a fair amount of valuable information from his stories and not only about theorems, ideas and trends in mathematics (e.g., "the main idea of contemporary topology is to exploit the simplifications that result in considering infinite-dimensional spaces" – while I always tried to replace this actual infinite dimensionality by a high, but finite, dimension of the manifold approximating the functional space).

It was he who told me about wicked characters that prevailed among mathematicians I would have to deal with. I am amazed when I think about it now, how serious he was about what is mildly put as "a breach in research ethics" (and what I have come to call shameless thievery, especially when it is taken from naive Russian authors who tend to talk about their ideas without publishing proofs or only publishing them in Russian journals).

At that time I was under the impression that Kolmogorov was one who dealt with these situations (which he constantly experienced) with regal indifference. But Rokhlin explained to me that it was just his good upbringing and self-control, but any such case insults every mathematician, and the more he tries not to show it, the more it shortens his life.

It was Rokhlin who taught me the subtle difference between the technical achievements in the Amerigo Vespucci style and Christopher Columbus-esque fundamentally new paradigms, which he valued more than any "sporting" achievements (like proving the Four Color or Fermat's Last Theorems).

"Some very gifted mathematicians", V.A. used to say, "are always on standby and as soon as a new idea appears they are able to appreciate it and manage to gain more dividends on it than the author". Only later did I find out how Vladimir Abramovich was right. There are countless results (especially, the ones belonging to Russian authors) that are stolen by the international gangs of epigoni and arrogated to their sidekicks. I was especially impressed by V.A.'s clear-sightedness when I caught in the act three of the standby specialists he specifically mentioned (I was not the victim). Clearly, the Americanized ethical system in the mathematical science nowadays does not provide for penalties for such crimes (in Russia one would not shake hands with these people).

I can see now that there is a certain expediency in this immoral system. The society benefits from the actions of go-getters who develop new ideas fast. Few people are aware that the prosperity of Bell Phone Company was built on the stolen invention. (The priority of Antonio Meucci, whose application "had been lost" by Western Telegraph and whom Bell promised to pay 20% of its dividends, was established by the Supreme Court in 1886, that is only after Meucci's patent application expired).

I could give more examples like this one from the world of mathematics, where there are no patents and where Meucci's part was played by Andronov, Petrovsky, Pontryagin, Kolmogorov, and the part of Bell – by very influential mathematicians of the West. I can just refer to a recent scandalous publication by B.Arratin, A.Barbour, S.Tavaré "Random Combinatorial Structures and Prime Factorization" in the Notices AMS (1997), 44:8, pp 903-910, which shamelessly expropriated an important theory created by Rokhlin's student A.M. Vershik.

I made it a point not to hush up similar crimes (the way other Russian mathematicians do in the interest of self-preservation and because they are financially dependent on mathematical community of the West) and not only because I am

always inclined to do so, but also to continue Rokhlin's legacy who thought that this situation was created only because the criminals know their deeds will go unpunished.

Indeed, nobody steals my own ideas (probably, out of fear), whereas almost all of my teachers, students, colleagues are being robbed and under-appreciated by the international mathematical community. Although, I heard that even the Nobel Prize Committee has become no more objective than the Fields one (it was brought to my attention by S. Smale even before the Nobel Prize for physics was awarded in 1997 for a discovery that was published in Russia according to official data about ten years before it was done by the laureate).

Rokhlin used to coach me that there are only two ways to avoid this kind of trouble. Either never tell anyone about your discovery until it is published (with all the corollaries and variations so that nobody can generalize anything at all), or tell everybody about it on every street corner so that they learn the results from the author himself.

To realize the second plan one had to attend international conferences and congresses a lot, which was impossible for us at the time. But I still chose the second way outlined by Rokhlin and would tell about my results at both Moscow and Leningrad mathematical societies meetings without waiting for anything to be published.

"From here it might seem," V.A. used to say, "that over there in the West they have more justice, and a scientist is judged according to his scientific achievements and not according to the party membership and ethnicity, as it is done here. But on closer examination the state of things proves to be even better here because there are two distinct groups that never mix together – ambitious social climbers and true mathematicians. We know who belongs to which group, and since there could not be any real scientists in the first group, we are able to evaluate true mathematicians in a more scientific and fair manner than in the West where scientific motives cannot be distinguished from ambition-driven ones."

I believe that these idyllic concepts (circa mid 1960s) need to be adjusted: our mathematical community is becoming more like international.

V.A. treated writing (of his own articles and the articles of his students) with ferocity and used to spend a disproportionately large amount of time polishing multitudes of them. He explained to me once that "all students belong to one of two categories: intelligent and not (and such a division has nothing to do with either their mathematical abilities or social background)".

According to Rokhlin, you only need to show an intelligent student once how to turn his structureless and incoherent babble into a nicely edited and well-structured text. At the beginning of such a text Rokhlin would usually place his own "Premium quality seal": "The terminology used in this article is that of differential topology".

In plain language these magic words meant, "The author of this article is V. A. Rokhlin's student".

The subsequent texts written by the same student will never need editing again. Simply put, he will not be able to write differently. Other equally gifted, but "not intelligent" mathematicians will write the second and the third article (that may contain remarkable results) in the same helpless way. All of them will have to be rewritten by the teacher, and if he stops doing it, the results will be lost

for mathematical community (until such time when someone may be willing to reformulate them properly).

When I complained to Vladimir Abramovich about the unreadable proofs of Ya .M. Eliashberg (I was a referee of his doctoral dissertation), V. A. responded: "I would never allow my student to publish such texts, but Eliashberg is not my student, but Gromov's. And you spoiled Misha yourself, when you wrote a positive referee report on his unreadable doctoral dissertation. I got tired of fighting with him and when I invited you to be a referee, I was hoping that you would smash his style. But you cut him some slack and Misha decided from then on that it was ok the way it was. And now you will have to put up with Eliashberg because of this!"

I have to mention that I could never come up with any counterarguments to any of Gromov's statements, while I was always able to find counterexamples to Eliashberg's ones (which were always very interesting). However, to fight back he would always modify his initial definitions for his theories, and that made them entirely incomprehensible for me as a result. After four or five of such modifications I still have no idea whether Eliashberg's proof of "Arnold's conjecture" (which in 1965 became a cornerstone of symplectic topology), described in his 1978 Syktyvkar work, is correct or not. Recently, Yasha promised to publish English translation of this work. Now, I hope, we will be able to find out if his proof was correct, the one that was 5 years ahead of the famous work of Conley and Zehnder (carried on by Floer, Chekanov, Gromov, Chaperon, Laudenbach, Sikorav, Hofer, Givental and many others).

The impact of Vladimir Abramovich on the writing style of works written in Russian in many different fields of mathematics (especially in differential and algebraic geometry, the singularity theory) was absolutely exceptional.

There is yet another Rokhlin's pedagogical theory, the true insight of which I have come to appreciate more and more over the years. According to Rokhlin, the teacher gives his student a gift he is not yet able to appreciate.

As a matter of fact, when the teacher sets the goal before his student, he performs a highly qualified job – pinpointing the main idea, communicating everything he knows about it, the significance of its meaning, and the results achieved in the field or even lack thereof, which is equally important. This work can be compared to that of a huntsman, leading the hunter to the right spot, or to the work of a guide in Himalayas.

A good teacher allows his student to find the solution oneself and creates an illusion that the student would achieve the same kind of success in the future, since he was able to overcome significant difficulties. Naturally, the student underestimates the significance of that advance preparation, which takes a completely different set of abilities and qualifications, and whose results he received as a gift from his teacher (in the form of the formulation of the problem and correct methodology).

Even if the student is an extremely gifted mathematician who is able to overcome significant difficulties, the outcome can often be tragic. The student is not able to produce any new results compared to his first outstanding achievement despite all his talent. He is not willing to produce less than brilliant work, and as a result he does nothing. ("Why do our young and talented mathematicians stop growing after having achieved their first success?" Kolmogorov would wonder, while he used to shower his students with his gifts.) If the student is smart enough (not as a mathematician, but as a human being), he starts studying seriously and

without chasing an instant success. First he gradually masters his field in general and after that he masters the art of formulation of problems. He either works out his own philosophy or finally realizes the significance of the gift he had received at the beginning and demands from his teacher (or starts looking somewhere else) for another gift of comparable value.

This is how Rokhlin explained the emergence of a multitude of schools of epigoni developing the ideas which were once new and fresh. (He named a whole list of Western and Russian mathematicians. Naturally, I am not going to repeat the names for fear of unintentionally hurting their feelings by forgetfully leaving out some of the names.)

Being an advocate of the purity of the Russian language, V.A. was very sensitive when it came to overusing of bureaucratic jargon. He would quote his teacher, A.I. Plesner (a German mathematician, who escaped Hitler's regime and moved to Russia) and who used to very diligently edit articles for the "Russian Mathematical Surveys" while saying "Your Russian Language grates *in* my ears." With all his brutal honesty Vladimir Abramovich was impeccably polite, especially with younger people and even more so with his students. His self-confidence and dignity prevented him from using any of many commonly used degrading methods of putting a person down. In spite of his passion for polemics, Rokhlin's noble and respectful attitude towards his opponent made him stand out, similar to the one that was admirable in Kolmogorov and Petrovsky.

I remember only one incident when V.A. who was accustomed to always winning, was forced to yield to a superior adversary. It all happened in Tsakhhadzor in 1969.

In that small town 2000 meters above sea level, high in the mountains of Armenia, used to be a training camp for the Olympic team, but at that time a Mathematics winter school was organized, where many fallen out of favor mathematicians gathered (mostly those who signed "The Letter of 99" in 1968 in support of Esenin-Volpin, a mathematician-dissident, who had been confined to a psychiatric hospital by the authorities).

In the morning, as always ignoring all the bans, I set off alone towards the slope for some skiing. As I was reaching the peak (about 3000 meters—now they have installed ski-lifts there, but at the time it was just a scenic spot in the wilderness with an amazing view of Lake Sevan), I got to the edge of the mountain and saw in about 20 meters a reddish boulder sticking out from the snow. It suddenly shook a bit and went rolling and not down the hill toward me, but up the slope. When I looked closely, I realized that it was a bear bolting from me.

It was almost lunch time. After a brisk descent through the virgin snow, I made to the cafeteria just in time – there waiting for me on the table was a steaming hot bowl of kharcho soup. I was about to tell Vladimir Abramovich about my adventure, when I nearly choked. I saw my bear through the glass wall of a pavilion, peacefully wobbling through the town. At that time they used to fatten up bears like piglets in Tsakhkadsor so that they could eat them later (they might still be doing it now).

It turned out that the bear was passionate about splashing out in the jet of water spurting out of the street water well pump. But he only knew how to do one of two things: he could either pull the lever with his paw and let others quench their

thirst or he could bathe and drink himself but in that case he needed somebody else to pull the pump lever for him.

After lunch Vladimir Abramovich and I were headed to a lecture and on our way there we met the bear at the water pump. The bear decided to entrust Rokhlin with the duty of pulling the lever. But V.A. was not aware of his habits and didn't seem to want to understand him. He tried to get rid of the bear using certain phrases that would (without insulting a person) convince him to give up any attempts of collaborating with V.A. Still the bear proved to be more persistent than Rokhlin. Accepting of his lack of insight, the bear grabbed Rokhlin's handsome white coat by its lapel with his teeth and dragged majestically elegant, in spite of what was happening, Vladimir Abramovich toward the water pump. (We had trouble explaining to Vladimir Abramovich what was it that the bear wanted from him). In any case, V.A. handled this incident with the bear with his usual both mathematical and non-mathematical undeterred aplomb. After all, he had gone through a lot more than that. (I also think that Rokhlin treated "The case of 99", which shook the whole mathematical hierarchy in Russia, much less tragically and more philosophically than the rest of us who remained hopeful until Khrushchev's "Thaw" collapsed in 1968.

I recall another conversation with Vladimir Abramovich, the subject of which he would return to again and again – his vision of the future of humanity. According to him, humanity is moving towards bureaucratization where an all-powerful bureaucratic apparatus will be suppressing everything alive and creative that still exists. According to him, this phenomenon is not exclusive to Russia, it is global, although this is an uneven process. Rokhlin thought that this process would be soon completed (in view of the fact that two-dimensional sphere is compact), and the Global Government will be created, which will realize the worst predictions of Zamyatin's "We" and Huxley's "Brave New World" on the global scale. Degenerating humanity lead by their worst representatives will democratically establish ochlocratic dictatorship, which will be suppressing everything out of the ordinary and will be mainly preoccupied with stopping progress, and, as a result, destruction of education and science (by means of dumbing down children from a very young age by watching TV, playing video and computer games).

Our times, the golden age of mathematics and science in general will then be considered an unprecedented highest point, the way we now think of Italian Renaissance Art, and Klein's "Lectures on Development of Mathematics in the Nineteenth Century" will read as Vasari's "The Lives of the Artists".

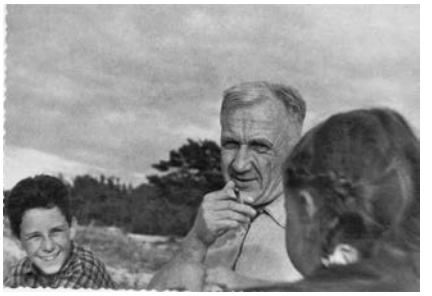
"I am glad I will not live to see that", concluded Rokhlin.

It is difficult to debate such predictions, however I would like to cite a similar prediction by Leo Tolstoy that has not quite become a reality. "The strength of the government lies in the people's ignorance, and government knows this, and will, therefore, always oppose true enlightenment".

A century has already passed, and the education has not been completely wiped out and it gives us a reason to look to the future with a touch of hope.

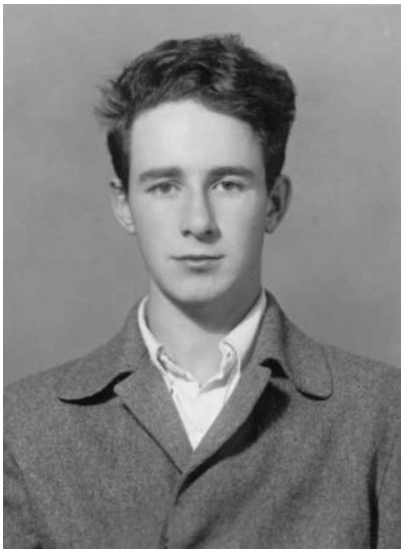
Photographs of V. I. Arnold

1940s–1970s



I. E. Tamm tells young Vladimir and Katya Arnold about "makhnovtsy", late 1940s; see Chapter 5.

In Riga, 1949.



An official photo, 1957.

In Palanga, 1953. Left to right: Dmitry Arnold (brother of V. Arnold), Lev Pereslegin (classmate of V. Arnold), Vladimir Arnold, Tatiyana Vainshtein (nee Mandelstam, second cousin of V. Arnold, granddaughter of L. I. Mandelstam, a prominent Soviet physicist).

Members of the Children's Learned Society (DNO), around 1948. Left to right: Andrei Novikov (brother of Sergei Novikov), Vladimir Arnold, Mikhail Zalkind, Dmitry Arnold (brother of V. Arnold), Sergei Novikov (well-known Russian mathematician), Oskar Krauze.

Outdoors, mid-1950s

At Nikolina Gora, with V. Rokhlin; see Chapter 7 for Arnold's memories about Rokhlin.

At Nikolina Gora, near Moscow.

On a hike, the 1960s.

In Kozha, the 1960s.

Painting, 1968.

With A. Kolmogorov (left), mid-1960s.

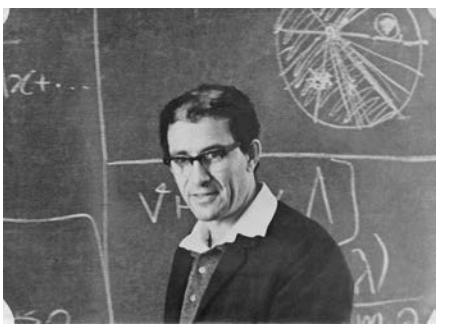
In Otepya, Estonia, the 1960s.

Ya. G. Sinai and V. Arnold in front of the main building of Moscow State University, 1963.

V. A. Rokhlin gives a lecture, 1960s; see Chapter 7 for Arnold's memories about Rokhlin.

President of the Moscow State University, I. G. Petrovsky (middle) with A. Kirillov and V. Arnold, around 1960.

Arnold with students at Kolmogorov's mathematical boarding school, the 1960s.



Lecturing in Syktyvkar, 1976; see Chapter 14 for Eliashberg's memories of Arnold.

CHAPTER 22

Vladimir Igorevich Arnold: A View from the Rear Bench

Sergei Yakovenko

Just ten days before reaching his 73th birthday our teacher, Vladimir Igorevich Arnold, or VIA, as we used to abbreviate his name between ourselves in correspondence, died in Paris from foudroyant peritonitis. The shock and feeling of evisceration was so strong that for several days those of us who were scattered around the globe were bombarded by phone calls and emails from those who happened to be in Paris or in Moscow. What? How could that happen? In rather good physical shape? Seemingly having fully recovered from the terrible bike accident that left him incapacitated for so long... The consciousness rejected the impossible. Yet in hours the news became a sad reality: VIA was indeed no more.

I felt a personal loss, though I could not pretend to be one of his intimate friends. I was not even his student in any sense of the word. I felt a spiritual loss: never again I would be able to learn from him anything beautiful and inspiring, curious or instructive, funny or mysterious. I felt a professional loss: the central pillar, around which so many events occurred and so many old friends and colleagues orbited, had fallen. The mathematical world as I knew would never be the same without VIA, without his encyclopedic knowledge and immense intuition, without his special charm.

The following is an edited version of the text which I wrote ten days later, on the birthday of Vladimir Igorevich.

Today, June 12, 2010, Vladimir Igorevich Arnold should have turned 73. Today, as many times on this day in the past years, I should have been writing a short informal "Happy Birthday" email that never was acknowledged; VIA was not known for wasting time on polite conversations, yet I knew he would have read it. If I were in Paris, I would call and drop by, as all of his students would do. Instead, today we are waiting for our Teacher to be laid to rest: The funeral in Moscow is scheduled for June 15.

The mere thought of Arnold being ill contradicts his personality as we remember him. All his life VIA projected strength, confidence, perfection, beauty, elegance. Physical, spiritual, mathematical, human. He was all motion, all burst. I remember him teaching the second-year class on Ordinary Differential Equations in the huge 16-24 hall of the Moscow University main tower building, during the 1977/8 academic year: VIA was then at the "Fields age", considered the prime

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age for mathematicians. At the beginning of each class, with the soundbite of the bell, he rushed in, his trademark briefcase in-hand, he started the first phrase of his lecture while still 3-4 meters from the blackboard. In a fraction of second his briefcase was thrown on the table, a piece of chalk appeared in his hand, and when the first phrase was completed, we already saw a carefully drawn picture on the blackboard and a few formulas written in his calligraphic handwriting near it. His lectures were practically impossible to write down, as impossible it is to record by a cell-phone a superb performance of your favorite music. Besides, it was very difficult to record the insight: As Arnold speaks, draws, writes, you suddenly see how different things are getting connected and the whole picture transpires through the initial fog. Fortunately, at that time his famous textbooks were already published; in these books he succeeded in doing the impossible and putting these revelations on paper.

In fact, it was probably my first hands-on experience with a working mathematician of such caliber, which forever left an imprint on my world view. Later encounters with VIA's peers (there was a unique constellation of great minds at this point in space-time) fascinated me but VIA always remained singular, even against such background. One should note, however, that his style of presentation of undergraduate subject traditionally considered as technical and boring, peppered with huge formulas and heavy computations, was not equally good for everybody. The feeling of crystal clarity that one got from VIA's exposition, was no substitute to the ability of restoring all missing "technical" details, and simplicity might well turn misleading. Many years later VIA mocked the "Bourbakist" way of spelling out mathematical statements in his famous quip, saying that the fact stated by Poincaré in the simple sentence "Pierre had washed his hands" in the formal Bourbakist rendering would sound like a description of the transition of Pierre from the set of dirty-handed to that of clean-handed at some moment in the past.[1]

Poignant and subtle, this quip does not obliterate the need for students to be able to translate "humanly understandable" phraseology into precise statements

[1]To the best of my memory, the first time this quip appeared was a footnote in the Russian (1986) edition of the survey paper *Catastrophe theory* (Russian), Current problems in mathematics. Fundamental directions, Vol. 5, 219–277, (Итоги Науки и Техники. Современные проблемы математики. Фундаментальные направления, ВИНИТИ), 1986. In a footnote on p. 233 VIA writes:

> К сожалетию, бесхитростные тексты Пуанкаре трудны для математиков, воспитанных на теории множеств. Пуанкаре сказал бы "Петя вымыл руки" там, где современный математик напишет просто: "Существует $t_1 < 0$ такое, что образ точки t_1 при естественном отображении $t \mapsto$Петя(t) принадлежит множеству грязноруких и такое $t_2 \in (t_1, 0]$, что Петя(t_2) принадлежит дополнению вышеуказанного множества".
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> Unfortunately, unsophisticated texts of Poincaré are thorny for mathematicians raised upon set theory. Poincaré would have said "Pierre has washed his hands" where a contemporary mathematician would simply write instead "There exists $t_1 < 0$ such that the image of t_1 by the natural map $t \mapsto$ Pierre(t) belongs to the set of dirty-handed, and $t_2 \in (t_1, 0]$ such that Pierre(t_2) belongs to the complement of the above set.

The Russian language has only one simple past tense and possessive pronouns are often omitted, thus the Russian phrase is more concise than its accurate English translation, making the contrast even sharper. But ironically exactly because of these grammar features the ridiculous "mathematical rendition" in fact adds to the initial Russian phrase the precision it missed.

equipped with all proper quantifiers, this is a task that not all were up to. Nevertheless, this could be considered as a part of VIA's teaching legacy: first the main and difficult things should be explained in simple terms, and only later the necessary technical details and subtleties should be addressed. Unfortunately, this approach goes against the mainstream of the current tradition of writing mathematical texts, where lemmas and preparatory technical stuff precedes the instances where they are required, and so lack motivation. VIA himself compared this "formal" style to cryptic biblical parables, which had to be expounded to disciples in seclusive meetings. Arnold's books are a unique example of mathematical literature where this traditional order is reversed. While keeping the trademark freestyle of presentation of the main issues, always accompanied by numerous drawings, he resorted to the fine print and "exercises for the reader" to deal with technical details. At one such instance he coined the phrase "It is easier to prove this statement singlehandedly than read a written proof" which indicates the level of detail, below which no lecturer should descend.

Later I started attending the famous Arnold's Seminar (with a capital "S"). It will certainly be described by many people who were both closer to VIA and have sharper pens, yet this phenomenon was so unique that no detail should fall into oblivion. The Seminar was scheduled so that people could attend it after the standard office hours, as many (probably, the majority) of the participants were not officially affiliated with the Moscow University. Arnold rushed in the room and took his permanent seat in the middle of the front row next to the blackboard. The seminar did not begin until VIA got from his briefcase a bunch of recent preprints and reprints and handed them out to the elder participants of the Seminar: "Vitya (to Vassiliev)! The author claims that he proved so-and-so, but I could not find any appearance of the contact structure in his computations. This simply cannot happen, we both know that it should be somewhere there!" (And in a couple of weeks Vitya would indeed return the manuscript to VIA with margins peppered by remarks explaining where the "missing" structure was concealed and showing how its explicit use may simplify the proof...). This "home assignment" could take quite a bit of time, yet at some moment Arnold opened his "school-like" copybook, entered the speaker's name and the title of the talk, and the Seminar began.

The choice of speakers and the titles, apparently, reflected the current interests of VIA himself; for me (at that time a 4th year undergraduate student) neither was telling, yet this was largely irrelevant since each Seminar was a one-man performance. A typical scenario was as follows. For the first 15-20 minutes the speaker talked "practically uninterrupted"—that's to say, no more than once in 1-2 minutes—when VIA asked questions seemingly technical or even bordering on chicanery. Gradually the exposition turned into an agitated conversation between the speaker and Arnold; this ping-pong match could last long enough for the rest of the audience to get completely lost. Then a culmination occurred: VIA jumped from his place to the blackboard and shouted "No, this is impossible to understand your way. The right picture should be as follows..." And then he explained in 5 minutes both the origin of the initial problem addressed by the speaker, its links and connections to other problems (and at that moment it became clear why Arnold invited this particular speaker to talk on this particular subject), and what the main result is proving (or disproving, or corroborating). In a few moments Arnold would explain how he would try to prove this result, and often the speaker,

changing colors from red to white, would nod in acquiescence... At such moments Arnold was literally shining from pleasure and suddenly would chuckle with his inimitable laughter, as a child who "just did it!".

This might well look like a derision of the speaker, yet it was not. The "retribution" could come instantly, when Arnold would start fantasizing about possible ramifications, generalizations and further developments that may come out of the result just learned. The speaker, regaining his balance by that time, could cut short these fantasies: "This corollary is indeed true, but the proof is by no means as simple as you think, VIA, for such and such reasons. And the generalization you suggest is simply wrong: just two weeks ago I constructed a counterexample" (of which the speaker did not plan to talk at all). At such moments VIA's excitement rose to a maximum: he would jump in again and start explaining why the speaker was wrong and what underwater rocks and unexpected phenomena manifest themselves in "so innocent a problem". It was these moments which justified attending the Seminar for two hard hours (sometimes longer). Even the youngest participants (like me) left the room exhausted yet with some clear mathematical message to take home.

This childish chuckle, instantly transforming the face of Arnold, in my eyes, reflected some part of his mathematical personality. He was very much like a prodigy child in Aladdin's treasure vault: enjoying mathematical reality in all its brilliance. Mathematical anecdotes mention great mathematicians whom examples only distracted from developing general theories. Arnold was the opposite: examples were the alpha and omega of his approach. Of course, it was impossible to look inside this beautiful mind, yet I have a feeling that he knew mathematical objects (small dimensional varieties, Lie groups, fundamental dynamical systems, ...) the way a zoologist knows and loves his bees, beasts, birds, etc. This was based on his tremendous erudition and, in turn, allowed him to see connections between seemingly very distant things. Probably, about any natural number less than one hundred, he remembered all mathematical results and constructions in which this number occurred.

One of the strongest impressions from the Seminar was the feeling of unity of Mathematics that literally radiated from VIA and the more senior participants. True, the similar feeling was also present on other seminars which I occasionally attended, but there it often was in the form of expanding horizons and relations with the branches not yet familiar to undergraduates. In Arnold's world, geometry of planar quadrics was connected with diophantine equations, Jordan form of matrices with the operator of derivative, functions of complex variable with probability. I remember that at a certain moment the difference between topological connectedness and arc connectedness appeared in the discussion of holomorphic dynamics: until then I was absolutely certain that examples illustrating the difference between these notions were specifically designed for exams in Calculus.

Discussing mathematics with Arnold was a unique experience. VIA was astoundingly sharp and quick-minded. I remember discussing with him a question indirectly related to one of his "Problems for the Seminar", on which I worked for quite some time. The problem was difficult (its complete solution took a further 25 years), and I tried to explain to VIA some partial results I had. The feeling was as if I was talking to a person who knows the answers to all questions; he seemed to be able to continue *my story* from any point, and in exactly the same way I did.

It was even embarrassing: all my efforts, weeks of banging my head against the wall could have been so easily spared, if only VIA would himself have decided to attack the problem! Only much later did I realize that Arnold instantly identified the key ideas from the very first phrases and then, with all his huge experience and intuition, he could indeed easily jump from hilltop to hilltop where I had to walk a difficult terrain in fog.

The impact of VIA on the generation of Moscow mathematicians, who are now approximately between 40 and 65, is enormous. His direct students exhibit a quasi-religious feeling towards him: no adjective (alone or in a combination) suffices to convey the impression he left. Lightning-fast thinking, sharp reaction, incredible intuition, ...—all attributes of a superhuman; he himself contributed to this image, stressing his physical skills like swimming, hiking, skiing, which also were well beyond "ordinary" capacity. Yet the child inside him was pretty much human: like many children, he loved to tease people, and many who didn't know him closely were understandably offended. For his students he often did (without saying) things that prove a deep personal involvement he felt towards them.

But even for those who "simply" happened to witness Arnold the Mathematician in action and enjoy the beauty and elegance of his view of the subject, the impact was catastrophic in the bifurcational sense of the word. At the time when I decided about the field of mathematical specialization, because of the unique atmosphere of the Moscow University in those days, the choice was tantalizing. Algebra and algebraic geometry with Yurii Ivanovich Manin, geometry or mathematical physics with Sergei Petrovich Novikov, probability and dynamical systems with Yakov Grigorievich Sinai, complex analysis with Anatoly Georgievich Vitushkin, Representations theory with Alexander Alexandrovich Kirillov-Sr., all in their prime, all bursting with energy, all doing beautiful mathematics. And of course, there was the proverbial figure of Israel Moiseevich Gelfand!

Instead I chose the subject which "before Arnold" many considered as boring, dull and non-inspirational; "A theorem on one property of one solution of one differential equation", quoting another of VIA's quips on "bad" Differential Equations. Since then I had not a single regret for falling in-love with such a wonderful part of Mathematics: its centrality and most diverse connections with almost all other areas is what I learned to enjoy, featuring a clear imprint of VIA's taste. My professional career was practically predetermined by the fact that it began in the epoch of Vladimir Igorevich Arnold illuminating my entrance to the universe of Mathematics.

The above memories (ranging from 1976 to the late 80s) describe what I would consider to be the absolute zenith of Arnold as mathematician, leader of a school and supreme commander of elite troops ready to follow him in attack on any mathematical fortress, fearless and ambitious. The subsequent changes in the country and the world obviously changed also many things in VIA's life. As I already mentioned, I did not belong to his most narrow circle of disciples and coworkers, myself having left Moscow in 1991, so necessarily the memories become stroboscopic and much more relying on hearsay rather than on my own first-hand experience.

VIA resisted the temptation to leave the USSR/Russia despite a desperate economic situation which rendered academic salaries practically nil. For some time a

partial solution for many was to look for visiting positions in the Western universities, work for several months a year abroad leaving families behind, and convert the accumulated salary into the source of modest subsistence, playing on the crazy exchange rates of the rouble at that time. However, such dynamic equilibrium was clearly unstable: some mathematicians from VIA's Seminar accepted permanent positions abroad, some gave up altogether. Quite a few exceptional people managed to arrange "permanent part-time positions" allowing them to spend one of the two semesters abroad, the other at home, in Moscow. Arnold resisted longer than many, but in 1993 he accepted such an offer from CEREMADE, a French CNRS unit at Université de Paris-Dauphine specializing in applied (sic!) mathematics. This has inevitably impacted the Moscow Seminar, although VIA himself made all efforts to ensure the continuity; e.g., he tried to re-create his Seminar in the spring semester to take place at exactly the same week day and time (Tuesdays, 16:20 till 18:00) in the École Normale Superieure.

However, the environment did matter, and the Paris Seminar did not rise to the place its Moscow prototype occupied in the mathematical world. The composition was different, the Parisian mathematical community did not reveal such acute interest in what was going on there, who knows what else went wrong... VIA, having a very dominant and assertive personality, felt the difference in the atmosphere and understandably grew more and more bitter about "the Western style" of doing mathematics. His criticism (very often more than well deserved) took forms which, apparently, many of his French colleagues had deemed offensive: for instance, he would never miss an opportunity to stress the fact that a certain problem, on which a respectable (and strong) French professor worked with only partial success, was "completely solved" by some young Moscow prodigy undergraduate. Both completeness of solution and the role Arnold himself could play in reaching it was conveniently stretched to produce infuriating effects. Another sad (in my view) crusade VIA launched about that time was against what he called "Bourbakism" and "pure mathematics". While the opposition to the formal axiomatic exposition of mathematical results was always characteristic of Arnold's trademark style (as I already mentioned), he gradually went overboard with ridiculing what he considered formalism and unnecessary abstractions. The mere names of Bourbaki and Hardy became anathema for Arnold, and the logical construction of solid foundations for future building (the trademark Bourbaki style) became the subject of ridicule more and more frequently. He went as far as to claim on several occasions that "there is no Mathematics, only a branch of Physics". Clearly, he did not mean these things literally, being himself a most subtle mathematician, but the chorus of jingoists of all stripes cheered these provocative statements, much to the chagrin of the genuine mathematical community.

After his tragic biking accident, VIA slipped more in this direction. Citing several rather anecdotic cases, he extended his (again, often perfectly legitimate and profound) criticism of the French high school and undergraduate education system to a blanket condemnation of the whole enterprise. Very often this was juxtaposed in VIA's diatribes to the (idealized at times) Soviet system of education; these writings were cheered by many, beyond all proportion. Eventually this side of his multifaceted activity took a very prominent place in the public perception of VIA: "Russian most-cited mathematician castigates the formal Western education system which perpetrates shallowness, and praises the Russian way of getting to

the heart of things!" VIA was made an icon of anti-Western rhetoric, completely ignoring the fact that he in fact was one of ecumenical figures in the modern science, universally recognized and respected by physicists, astronomers, topologists, algebraists, analysts of various traditions of all countries... It would be very sad if the monochromatic image of an iconoclast would be perpetuated, shading the uniqueness of VIA in his ability to get to the core of things in all their diversity. He himself could learn and teach this way, only a few could follow in his footsteps.

According to Arnold, the last words of Isaac Barrow, the adviser of Isaac Newton, were "Oh Lord! Soon I will know solutions to all differential equations". Today we know how naïve this wish was, yet more important things stay forever. Vladimir Igorevich, I wish you to know that the seeds you planted all your life will yield hundredfold harvests. Any other outcome would be unfair, ugly, and hence, simply, wrong, as the truth is always beautiful...