

Preface

Dear reader!

The book you are currently holding in your hands is an invitation for the pupils in grades 6 and 7 to take part in a Mathematical Festival. It traditionally takes part every year on one of the Sundays in February in the Main building of the Moscow State University on Sparrow Hills (Vorob'evy Gory). Hundreds of schoolchildren attend on this day in order to solve interesting problems.

WHAT IS A MATHEMATICAL FESTIVAL AND HOW DOES IT WORK?

The festival begins at 10:00 (of course, it's better to arrive 10–15 minutes early). The participants are firstly seated in lecture halls and the “rules of the game” are explained to them, after all this is the first mathematical competition ever for many of them.

The rules are pretty simple. You have to try to solve as many problems as possible, and do this alone—you are not allowed to talk to your neighbors. (You have to play fair!)

Write down the solutions neatly on special forms, so the people checking them could read your solutions and understand them! It's important that you write down the solution (explanation) as well as the answer to each question. Don't be afraid to start writing down the solution if everything seems clear, but you don't know where to start. The problems in an Olympiad are unusual, and unlike school tests there are no specific rules of presentation of the solutions. Apart from this, no one will lower your grade if you cross things out, and small slips of the pen will also be forgiven (if they don't affect the solution). The most important thing is that the idea is clear and your thoughts are easy to understand. Even if you haven't managed to solve the problem, write down what you have managed to come up with—it may be that you are just one step away from your goal, and this will certainly be

acknowledged. Remember that a correct answer alone is often worth less than a good solution with a mistake at the end.

If you're stuck on a problem, it is best to move onto the next one and then return to the previous one again. It's always a good idea to write down any solution straight away, and then reread it later.

You have 2 hours to solve the problems. While the children are solving problems, the parents are meeting the representatives of the organizing committee, the governing body of the MSU Department of Mathematics and Mechanics, leaders of after-school math circles, teachers of the leading city schools.

Then there is a break, during which you can listen to an analysis of the problems, attend an interesting lecture on mathematics and have a snack.

A cultural program begins at 14:00 (usually it's a cartoon showing). The award ceremony takes place at around 17:00-17:30. After the award ceremony there is a display of Olympiad papers.

Information on when the Mathematical Festival will take place, problems from previous years and statistics are always available on the website of the Moscow Center for Continuous Mathematical Education (www.mccme.ru).

We are looking forward to seeing you at the next Mathematical Festival and the Moscow Mathematical Olympiad!

A HISTORY OF THE FESTIVAL

The first Mathematical Festival took place in 1990, initiated by the teachers of the mathematics club at the MSU (The Lesser School of Maths and Mechanics), Dima Botin (to whose memory this book is dedicated), Sasha Spivak and the author of these lines. There were around 200 kids at the first celebration, mostly from math circles. From 1990 onwards the Mathematical Festival became an annual event. The number of participants grew steadily. In 2004 there were already 2000 of them. Many participants came from other cities. In 1994 the Festival became part of the Moscow Mathematical Olympiad.

In these years it turned from a small competition between members of math circles into a brilliant festival for hundreds of kids. After experiencing the atmosphere of mathematics, many of its participants (and not only the winners!) successfully continued to take part in mathematical circles, went to specialised schools and finished leading universities, especially the MSU.

The Mathematical Festival is a joint effort of hundreds of enthusiasts: students of the Department of Mathematics and Mechanics, math circle leaders, university professors, teachers and older students of the

leading mathematical high schools. A great thank you to all of them. There would be no Festival without them. . . The size of this book won't let me list all those who have given part of their soul to the Mathematical Festival in the past 20 years, so I will only mention those who have made a great contribution to the work of the organizing committee and the methodological committee: L. D. Altschuler, N. N. Andreev, V. D. Arnold, A. D. Blinkov, D. Botin, V. Bugaenko, P. V. Chulkov, B. M. Davidovich, S. A. Doritschenko, R. Fedorov, V. Furin, T. Galina, B. P. Geidman, T. Golenishcheva-Kutuzova, D. Grigorenko, V. Gurovitz, Yu. Herman, E. Yu. Ivanova, A. N. Karpov, A. Khachatryan, V. P. Khovanski, V. Kleptsyn, A. K. Kovaldzhii, V. Kryukov, Yu. Kudryashov, R. Kusnets, N. Kulakova, A. Kulygin, S. Markelov, A. Mityagin, M. Panov, M. Potanin, V. Radionov, I. Raskina, S. Shalunov, I. F. Sharygin, A. Shen, A. V. Spivak, V. V. Yashchenko.

The Festival would not have been possible without the support from the governing body and the staff of the Moscow State University (above all the chancellor and the Department of Mathematics and Mechanics): on this day most of the Vorob'evy Gory complex of the MSU is at the disposal of the participants of the competition.

From 1994, the Mathematical Festival has been supported by the Department of Education and the Moscow Institute of Open Education. For the past few year this work takes place within the framework of the program "Talented Children" in Moscow.

And, last but not least, the Mathematical Festival would not have any participants, if the teachers, club leaders and parents didn't help the children study mathematics.

ABOUT THIS BOOK

This book is based on the third (extended) Russian edition published in 2009. The first and second editions came out in 1998 and 2005.

In the first section of the book you will find problems from the Mathematical Festivals in 1990-2011. You can simply solve those problems that you like the look of, or you could sometimes even have a practice Olympiad — try to solve as many problems from one Olympiad year as possible in 2 hours. Usually (but not always!) the first two problems are slightly easier and the last ones are a bit harder, and all of the problems have different topics, so solving 2-3 problems is already not a bad achievement! Sometimes (especially in the case of the problems from the early festivals, which were mostly attended by kids from math clubs) you may find problems requiring knowledge that is technically beyond the school syllabus for the given year. Although nowadays,

with an ever-increasing number of alternative text books, it's difficult to determine what exactly this school program consists of. Yet if you have enough intuition and a quick mind, it's always possible to find a way to solve them!

After you have solved the problem, take a look in the second section of the book and check your answer. It's also highly advisable to check your solution against the one provided in the fourth section of the book — apart from checking that your solution is correct (even if you have the correct answer, your solution might be wrong!), you may also find out different ways of approaching the problem or read interesting comments.

If you can't get your head around a problem, it's a good idea to take a look in the third section of the book — here you may find a hint or even an idea for a solution. But don't hurry — don't deny yourself the joy of a small mathematical discovery! A teacher may use these hints to help his students during an after-school math club (but not at an Olympiad!).

In the thematic index at the end of the book the problems are classified by theme, according to the main ideas in their solutions. This will allow a schoolchild to practice specific solution methods, and the teacher to select problems for a mathematical circle.

On the website of the Moscow Center for Continuous Mathematical Education (www.mccme.ru) and on the website of the "Problems" project (www.problems.ru) you can find many other problems, interesting materials for math circles clubs and individual study, including the problems and solutions from most of the mathematical competitions (www.mccme.ru/olympiads), wonderful texts ranging from the classical *Arithmetic* by L. F. Magnitsky to recent books (www.mccme.ru/ilib), and archives of *Kvant* magazine (kvant.mccme.ru).

ACKNOWLEDGEMENTS

Apart from the archives of the author himself, the materials used during the preparation of this book include collections of problems and solutions that are annually published by the organizing committee, the book *Moscow Mathematical Olympiads 60 Years Later* (compiled by A. Ya. Kanel-Belov, A. K. Kovaldzhi, edited by Yu. S. Ilyashenko and V. M. Tikhomirova) and materials provided by D. Botin, V. Bugaenko, R. Fedorov, R. K. Kovaldzhi, A. V. Spivak. The manuscript was carefully read by V. D. Arnold, A. Bokhenek, A. Kapliev, T. Karavaeva, Yu. Kudryashov, A. V. Semenov, M. Tsvetkov, D. and M. Veltishchevs, V. V. Yashchenko, who provided many useful comments.

A special thank you to V. Radionov, who not only prepared the original mock-up of the book and the illustrations, but also corrected a number of inaccuracies.

I. Yashchenko