Abstract

We consider the semilinear focusing wave equation

$$\partial_{tt}u - \Delta u - u|u|^{p-1} = 0$$

in large dimensions $d \geq 11$ and in the radial case. For a range of energy supercritical nonlinearities $p > p(d) > 1 + \frac{4}{d-2}$, for each integer large enough $\ell > \alpha(d,p) > 2$, we construct a Lipschitz manifold of codimension $\ell - 1$ of solutions blowing up in finite time T by concentrating the soliton (stationnary state) profile:

$$u(t,r) \sim \frac{1}{\lambda(t)^{\frac{2}{p-1}}} Q\left(\frac{r}{\lambda(t)}\right)$$

at the quantized blow up rate :

$$\lambda(t) \sim c_u (T-t)^{\frac{\ell}{\alpha}}.$$

The solutions can be chosen C^∞ and compactly supported. In that case the blow up is of type II i.e all norms below scaling remain bounded

$$\limsup_{t \uparrow T} \|\nabla^s u(t), \nabla^{s-1}\partial_t u(t)\|_{L^2} < +\infty \quad \text{for} \quad 1 \leq s < s_c = \frac{d}{2} - \frac{2}{p-1}.$$

Our analysis adapts the robust energy method developed for the study of energy critical bubbles by Merle-Raphaël-Rodnianski, Raphaël-Rodnianski and Raphaël-Schweyer, the study of this issue for the supercritical semilinear heat equation done by Herrero-Velázquez, Matano-Merle and Mizoguchi, and the analogous result for the energy supercritical Schrödinger equation by Merle-Raphaël-Rodnianski.

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