Abstract

We consider the semilinear focusing wave equation
\[ \partial_{tt} u - \Delta u - u|u|^{p-1} = 0 \]
in large dimensions \( d \geq 11 \) and in the radial case. For a range of energy supercritical nonlinearities \( p > p(d) > 1 + \frac{2}{d+2} \), for each integer large enough \( \ell > \alpha(d,p) > 2 \), we construct a Lipschitz manifold of codimension \( \ell - 1 \) of solutions blowing up in finite time \( T \) by concentrating the soliton (stationnary state) profile:
\[ u(t,r) \sim \frac{1}{\lambda(t)^{\frac{2}{p-1}}} Q \left( \frac{r}{\lambda(t)} \right) \]
at the quantized blow up rate:
\[ \lambda(t) \sim c_u (T - t)^{\frac{\ell}{\alpha}}. \]
The solutions can be chosen \( C^\infty \) and compactly supported. In that case the blow up is of type II i.e all norms below scaling remain bounded
\[ \limsup_{t \uparrow T} \| \nabla^s u(t), \nabla^{s-1} \partial_t u(t) \|_{L^2} < +\infty \quad \text{for} \quad 1 \leq s < s_c = \frac{d}{2} - \frac{2}{p-1}. \]
Our analysis adapts the robust energy method developed for the study of energy critical bubbles by Merle-Raphaël-Rodnianski, Raphaël-Rodnianski and Raphaël-Schwayer, the study of this issue for the supercritical semilinear heat equation done by Herrero-Velázquez, Matano-Merle and Mizoguchi, and the analogous result for the energy supercritical Schrödinger equation by Merle-Raphaël-Rodnianski.

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