

Foreword

Contemporary financial mathematics and the associated theory of actuarial calculations in insurance have reached such a level of mathematical complexity and abstraction that it is impossible to present a maximally rigorous exposition of them in brief form without detriment to either mathematical correctness or breadth of illumination of the subject. However, only the general theory gives rise to a fundamental approach to the problem on which a practically acceptable methodology for solving it can be based. Contact with practitioners of financial and insurance businesses has shown that these pragmatists are interested first and foremost in the practical realization of concrete models.

Thus arose the plan for this book: to present key and mathematically very complex results in the contemporary theory of hedging and investment at the brink of mathematical correctness, and to show as rigorously as possible how this general methodology can be interpreted in concrete models of financial markets.

This approach, which broadens the spectrum of readers beyond the narrow circle of specialists in stochastic analysis, was the basis for the advanced courses of lectures on financial and actuarial mathematics which I delivered at the Mechanics and Mathematics Department of Moscow State University in 1997–2000 and at the Laboratory of Actuarial Mathematics of Copenhagen University in 1998.

The problems touched upon here are being worked on very intensively, as reflected by the appearance of a whole spectrum of publications, of which I single out the 1999 monograph *Essentials of stochastic finance: facts, models, theory* by my teacher A. N. Shiryaev. It is a fundamental encyclopedia in this area.

Nevertheless, the monograph literature does not yet adequately reflect incomplete markets, markets with constraints on the strategy and structure of the model, “imperfect” forms of hedging (quantile and mean-variance), the “convergence” of financial and actuarial mathematics, and so on. This whole area, which forms a methodological basis for modern quantitative calculations in finance and insurance, is represented in the present book, and as a rule the chosen form of presentation of the material generally goes “from the general to the particular.” The above as a whole distinguishes the book from other publications in the area.

After writing the first three chapters, I enlisted the services of my students and colleagues S. N. Volkov and M. L. Nechaev for work on the book. They had prepared the first Russian dissertations devoted entirely to contemporary financial mathematics at the Steklov Institute of Mathematics of the Russian Academy of Sciences. Chapters 4–8 were written jointly with them. The exposition of the concluding Chapter 9 follows my paper *On the unity of quantitative methods of calculations in finance and insurance* (preprint no. 5, Actuarial-Financial Center, Moscow, 2000).

I would like to believe that this book will find an audience among both theoreticians and practitioners in finance and insurance and that it can serve as a basis for modern courses in actuarial and financial mathematics and quantitative risk management for university specializations in the direction of mathematical economics.

A. V. Mel'nikov