Foreword

As a measure of how a surface curves, we have the Gaussian curvature and the mean curvature. The Gaussian curvature, an intrinsic quantity, became one of the foundations for the development of Riemannian geometry. In contrast, the mean curvature is an extrinsic quantity which measures how the surface lies in space. Since the mean curvature is related to the character of the surface of a material body, it is deeply related to other sciences.

The mean curvature is defined as the arithmetic mean of the two principal curvatures at each point of a surface. A surface whose mean curvature is 0 at each point is called a minimal surface, and an area-minimizing surface is an important example of this. Minimal surfaces continue to be studied actively and have deep relations to analysis, especially the theory of functions.

A surface whose mean curvature is constant but not equal to 0 is obtained when we minimize the area of a surface while preserving its volume; the sphere is a trivial example and the constant mean curvature torus discovered by H. Wente in 1984 gave geometers a powerful incentive to study such surfaces. Subsequently, many constant mean curvature surfaces were discovered using a variety of techniques.

In this book, we aim to explain various examples of constant mean curvature surfaces and the techniques for studying them.

In Chapter 1, we define smooth surfaces and explain the basic notions of differential geometry that are necessary for the local study of surfaces. In Chapter 2, we explain the mathematical and physical meaning of the mean curvature.

In Chapter 3, we consider surfaces of revolution having constant mean curvature. Although the results in this chapter were obtained in the mid-nineteenth century by C. Delaunay, they are necessary to understand the more sophisticated modern examples. Next, we investigate constant mean curvature surfaces invariant by helicoidal motions in Chapter 4 and show that such surfaces are obtained by deforming Delaunay surfaces isometrically.

In Chapter 5, we define general surfaces. As a result, we can study global properties of surfaces, and we discuss the stability of constant mean curvature surfaces.

In Chapter 6, we introduce a closed constant mean curvature surface which is not the sphere. It is a torus, topologically, but quite different from an ordinary doughnut-shaped one.

Chapter 7 is an explanation of the basic methods used to study the general theory of complete constant mean curvature surfaces. In particular, we obtain the balancing formula, which controls behavior of infinity of complete surfaces.

In Chapter 8, we introduce the study of constant mean curvature surfaces via their Gauss maps. Harmonic maps from Riemann surfaces to the Riemann sphere appear here. We mention a representation formula for constant mean curvature surfaces in this chapter.

In Chapter 9, we explain existence theorems for constant mean curvature surfaces with or without boundary. Moreover, using material from recent studies, we explain discrete constant mean curvature surfaces and a technique used to construct constant mean curvature surfaces.

In the Appendix, we explain calculations which were too long to put in the text, some theorems used in the latter part of this book, the maximum principle for elliptic partial differential inequalities, the Alexandrov reflection technique, and so on. Moreover, we present the *Mathematica*® programs written by the author that are used for making the figures in this book.

We assume that the reader has some knowledge of calculus (at least as far as Green's theorem), linear algebra, and elemental differential geometry. In addition, some acquaintance with manifold theory, such as differential forms and elementary topology, is desirable for reading Chapter 5.

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Sendai

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Regarding the second edition: We corrected some mistakes in the first edition and replaced Figure 3.8. Moreover, we improved and revised the programs in Appendix B (Figure 3.4, Figure 3.7, Figure 3.8, and Figure 3.9) so that the figures can be drawn more quickly. The author expresses his appreciation to Professor Takashi Ogata of Yamagata University, Professor Yusuke Sakane of Osaka University, and Shinya Hirakawa, graduate student of Tohoku University, for their assistance in making the revisions.

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