

Author's Preface

The present volume, *Essentials of Stochastic Processes*, is an English translation of my book written in Japanese and issued by Iwanami Shoten in 1957 in two parts: Stochastic Processes I (from Chapter 1 to 3) and II (from Chapter 4 to 5). In this work, I provide a unified and comprehensive account of additive processes (or Lévy processes), stationary processes, and Markov processes, which remain to this day the three most important classes of stochastic processes. I had sent the Japanese original at the time of its publication to Eugene B. Dynkin, and I was very pleased to see A. D. Wentzell's Russian translation published in 1960 (Part I) and 1963 (Part II). I am also grateful to Dynkin for editing the translation and adding some important clarification footnotes. In 1959 Shizuo Kakutani at Yale University, noting the significance of my description of the one-dimensional diffusions, advised Yuji Ito, then one of his graduate students, to produce a translation of Part II into English, which was distributed among a limited circle of mathematicians around Yale University as a typewritten mimeograph. On the occasion of my receiving the Kyoto Prize in 1998, Shinzo Watanabe and Masatoshi Fukushima encouraged me to have the entire 1957 book translated into English and published by the American Mathematical Society. Yuji Ito graciously agreed to take on this arduous task and revisited his earlier partial translation, not only adding Part I, but also fully revising his original translation of Part II.

Although almost half a century has passed since the initial publication in Japanese, I hope there is enough of value in this work to merit its publication in English at this time. It should be noted that some detailed introductions to additive processes and Markov processes are given in two of my lecture notes published later on:

- *Lectures on Stochastic Processes*, Tata Institute of Fundamental Research, Bombay, 1960.
- *Stochastic Processes*, edited by Ole E. Barndorff-Nielsen and Ken-iti Sato, Springer, 2004 (originally published as Lecture Notes from Aarhus University in 1969).

However, the present volume is the only one among my English textbooks that includes an introduction to stationary processes.

Chapter 5 is devoted to the one-dimensional diffusion theory which is important as a basic prototype of the study of Markov processes. This chapter starts with a presentation of the local generator of a one-dimensional diffusion process as a generalized second order differential operator discovered by William Feller several years before I wrote this book. It then proceeds to a detailed description of the

boundary behaviors of the solutions of the associated homogeneous and inhomogeneous equations in an analytical way, followed by their probabilistic implications on the path properties of the diffusion near the boundaries.

My lecture notes from the Tata Institute mentioned above contain another detailed explanation of the Feller local generator. Section 4.6 of my joint book with H. P. McKean (*Diffusion Processes and Their Sample Paths*, Springer, 1965; in *Classics in Mathematics*, Springer, 1996) also exhibits the boundary behaviors with some probabilistic proof, while sections 5.12, 5.13, and 5.14 of the present volume are readily understood even by readers unfamiliar with probability theory.

When I wrote the original Japanese version of this book, the real study of stochastic processes had just begun, and not much related literature was available as noted in the Postscript. In the five decades since then, there have been significant developments in the theory of stochastic processes with many important subsequent publications, some of which are listed in the Preface to the Original and the Foreword by the Editors in the above-mentioned *Stochastic Processes* published in 2004 based on my Aarhus Lecture Notes.

I am very much indebted to those who have helped me bring this translation project to a successful completion. My gratitude, first and foremost, goes to Yuji Ito for the precise yet elegant translation which far exceeded my expectations, and I sincerely wish to thank him once again for his time and efforts. My thanks are due to M. Fukushima, K. Ichihara, and S. Watanabe for the meticulous care they took in proof-reading and editing the translated manuscript. This English version is in many ways superior to the original in that it eliminates minor inconsistencies and updates some of the discussion. In particular, the original version in Japanese, written when I had just started my work on paths in Markov processes, contains discussions of the general theory in Chapters 4 and 5 that are in hindsight somewhat unclear and misleading. I am grateful to M. Fukushima and S. Watanabe for suggesting the appropriate amendments in these chapters.

In view of the fact that Professor Shizuo Kakutani had first suggested, shortly after its Japanese publication in 1957, that my book be translated into English, I had hoped to be able to finally present him with this English version published by the American Mathematical Society. It was with great sadness that I learned of his passing away in the summer of 2004 in New Haven. In order to express my deep respect and admiration for his teaching and his important contributions to mathematics, I wish to dedicate this book to the late Professor Shizuo Kakutani.

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K. Itô