Preface

In this edition we corrected some errors and misprints and added references and footnotes related to recent achievements in the topics considered in the original edition. A survey of the results obtained since 1970 is in Appendix B, after the authors' Notes. There is also a separate list of references added to the English edition, which follows the original Bibliography. These references are numbered consecutively, to distinguish them from those in the main Bibliography.

Preface to the Russian Edition

The origins of the value distribution theory of meromorphic functions go back to the classical theorems of Sokhotskii–Casorati (1868), Weierstrass (1876), and Picard (1879). In the last decade of the 19th and the first two decades of the 20th centuries, these theorems underwent further development through investigations on the zero distribution of entire functions carried out mainly by the French school (Hadamard, Borel, Valiron, and others). The analytical machinery intrinsically connected with meromorphic functions was built in the 1920s by the Finnish mathematician Rolf Nevanlinna. After his work, the value distribution theory acquired, in some way, a complete form. The main classical results of the theory of entire functions have been included in Nevanlinna's theory in a natural way.

The results of Nevanlinna's theory, which, at the present time, can be regarded as classical, are considered in the first part of this book, consisting of Chapters 1–3 and Sections 1–3 of Chapter 4. Our aim here is to introduce beginners to the subject, and therefore our exposition does not skip details. We foresee reproaches of more qualified readers in this respect.

While the first part of this monograph treats mostly results obtained before the 1950s, the remainder is dedicated to modern research.

In spite of a certain completeness of the value distribution theory, the study of even most classical problems has not been brought to an end. On the contrary, it becomes more and more extensive. Many important problems remain open, and new problems arise in the process of further investigation.

The achievements are so essential and diverse that they admit the free coexistence of several monographs on the value distribution theory. In any case, the reader who is acquainted with Hayman's book [**Hay64**], will not find any substantial intersection with our book except for the classical theorems which had been known as early as the 1930s.

The value distribution theory of meromorphic functions occupies one of the central places in Complex Analysis. Extensive research is devoted to its connections with other areas of mathematics (topology, differential geometry, measure theory, potential theory, and others), to extention of its results to larger classes of functions (meromorphic functions in arbitrary plane regions and Riemann surfaces, algebroid functions, functions of several variables, meromorphic curves), and also to its applications, mainly in the analytic theory of differential equations.

In this book, the related topics are left aside, and the main attention is concentrated on the internal problems of the value distribution theory, which include

the following:

(i) To what extent are the main results of Nevanlinna's theory definitive and cannot be improved further?

(ii) What properties of Picard's exceptional values are preserved for a wider class of exceptional values considered in the value distribution theory?

(iii) What are the connections between Nevanlinna's characteristics and other quantities that describe asymptotic properties of entire and meromorphic functions?

(iv) Also, we study asymptotic properties and value distribution of meromorphic functions that belong to some special classes which are, on one hand, sufficiently narrow to give new information not implied by general theorems and, on the other hand, sufficiently wide to be of interest for the general theory.

(v) Moreover, we study the value distribution with respect to arguments (not only with respect to moduli as in the classical Nevanlinna theory).

We pay much attention to the examples of functions with "pathological" properties. Without them, the reader would get a restricted image of the theory. Examples of functions with unusual properties play in the value distribution theory an important role, the same way counterexamples do in Real Analysis.

In conclusion, we indicate terminology, conventions, and notation used throughout the book without further notice.

Meromorphic functions are functions meromorphic in the complex plane, unless explicitly stated otherwise. Uniform (absolute or usual) convergence of a series or of an infinite product is understood as the corresponding convergence of its remainder. Thus, the behavior of finitely many terms does not affect the convergence or its type.

If we consider a continuous function with a removable discontinuity, we assume that this discontinuity is removed. For example, $f(0) = 1$ for $f(\lambda) = \pi\lambda \cot \pi\lambda$.

By $[\alpha]$ we denote the integer part of the number α. Square brackets are also used to separate expressions in formulas if this cannot cause any confusion. We also use square brackets for references. We denote by α^+ the number $(|\alpha| + \alpha)/2$.

The closure of the angle $\{\alpha < \arg z < \beta\}$ in the complex plane is denoted by $\{\alpha \leq \arg z \leq \beta\}$. Thus $0 \in \{\alpha \leq \arg z \leq \beta\}$.

Writing $\sum_k a_k$ or $\prod_k a_k$ we do not exclude the case where the sequence $\{a_k\}$ is finite or empty. In the latter case we assume that $\sum_k a_k = 0$ and $\prod_k a_k = 0$.

In each chapter, the formulas are numbered within sections. Thus, the reference (4.20) means formula (4.20) of the same chapter. Similar numeration is used for theorems. When we refer to a formula or a theorem from another chapter, we include the chapter number.

The facts related to the history of the problems under consideration, along with bibliographical references, are placed in the Notes at the end of the book. References to the Bibliography in the text are given only to results that lie beyond the scope of standard graduate courses in Real and Complex Analysis.

We note that the results of Sections 3 and 5 of Chapter 1 and of Section 3 of Chapter 3 are used in Chapter 6 only, and can be skipped in the first reading.

Preliminary versions of Sections 7 and 8 of Chapter 1; Chapter 2; Sections 1 and 2 of Chapter 3; Sections 1–4 and 6 of Chapter 4; Sections 1 and 2 of Chapter 5; Chapter 7; and Appendix A have been written by A. Goldberg. Sections 1–5 of Chapter 1; Section 3 of Chapter 3; Section 5 of Chapter 4; Sections 3–6 of Chapter 5; and Chapter 6 have been written by I. Ostrovskii.

We express our deep gratitude to Nikolai Govorov, Larisa Kudina, Victor Petrenko, David Potyagailo, Elena Sergienko, and Stanislav Tushkanov for their help with the book at various stages. Our special thanks to our editor Naum Landkof.

<div align="center">Anatoly A. Goldberg and Iossif V. Ostrovskii</div>