

Contents

Preface	xi
Preface to the Russian Edition	xiii
Chapter 1. Characteristics of the Behavior of a Meromorphic Function and the First Fundamental Theorem	1
1. Auxiliary results	1
2. The Nevanlinna, Poisson–Jensen, and Shimizu–Ahlfors formulas	6
3. The Carleman and Levin formulas	10
4. The Nevanlinna characteristics. The first fundamental theorem of the value distribution theory	13
5. The angular Nevanlinna characteristics, Tsuji characteristics, and related analogs of the first fundamental theorem	23
6. Relations between the characteristics	28
7. Connection between the growth of the modulus of a meromorphic function and the growth of its characteristic $T(r, f)$	36
Chapter 2. Meromorphic Functions of Finite Order	43
1. The growth scale for meromorphic functions	43
2. Proximate order	49
3. Infinite products	54
4. Hadamard’s and Lindelöf’s theorems	57
5. Examples	66
Chapter 3. The Second Fundamental Theorem	87
1. Lemma on the logarithmic derivative	87
2. The Nevanlinna second fundamental theorem	95
3. Analogs of the second fundamental theorem for angular Nevanlinna characteristics and for Tsuji characteristics	104
Chapter 4. Deficient Values	113
1. Exceptional values	113
2. The set of Valiron exceptional values	116
3. Deficiency relation	121
4. The structure of the set of deficient values	122
5. Entire functions with infinitely many deficient values	132
6. Stability of deficiencies	149
Chapter 5. Asymptotic Properties of Meromorphic Functions and Deficiencies	171
1. Asymptotic values	171
2. Non-asymptotic deficient values	180

3. Meromorphic functions of order less than $1/2$	201
4. Bounds for the sum of two deficiencies and related problems	218
5. The growth of a meromorphic function with exceptional values	237
6. Meromorphic functions representable by series of simple fractions	251
Chapter 6. Value Distribution with Respect to the Arguments	259
1. Meromorphic functions with separated poles and zeros	259
2. Meromorphic functions with poles and zeros located close to a system of rays	276
3. Proofs of Theorems 2.3 and 2.7	294
4. Meromorphic functions with poles and zeros in small angles	306
5. Entire functions with derivatives vanishing only near the real axis	317
Chapter 7. Applications of Riemann Surfaces to Value Distribution	331
1. Geometric meaning of deficient and index values	331
2. Quasiconformal mappings	335
3. The Teichmüller theorem	345
4. Riemann surfaces of the class F_q and line complexes	352
5. Statement of the inverse problem of the value distribution theory. Riemann surfaces with finitely many logarithmic ends	377
6. Almost periodic ends	385
7. Riemann surfaces with finitely many almost periodic ends	403
8. The inverse problem of the value distribution theory	411
Appendix A. On the Magnitude of Type for an Entire Function	429
Notes	439
Chapter 1	439
Chapter 2	439
Chapter 3	441
Chapter 4	441
Chapter 5	442
Chapter 6	446
Chapter 7	449
Appendix B. A Survey of Some Results after 1970 (by A. Eremenko and J. K. Langley)	453
Chapter 1	453
Chapter 2	454
Chapter 3	455
Chapter 4	458
Chapter 5	459
Chapter 6	462
Chapter 7	463
Appendix A	464
Bibliography	467
References Added to the English Edition	475
Author Index	481

CONTENTS

ix

Subject Index

485

Notation Index

487