

A History of Prizes in Mathematics

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1. Introduction

Problems have long been regarded as the life of mathematics. A good problem focuses attention on something mathematicians would like to know but presently do not. This missing knowledge might be eminently practical, it might be wanted entirely for its own sake, its absence might signal a weakness of existing theory — there are many reasons for posing problems. A good problem is one that defies existing methods, for reasons that may or may not be clear, but whose solution promises a real advance in our knowledge.

In this respect the famous three classical problems of Greek mathematics are exemplary. The first of these asks for the construction of a cube twice the volume of a given cube. The second asks for a method of trisecting any given angle, and the third for the construction of a square equal in area to a given circle.¹ Because Euclid, in his *Elements*, used only straight edge and circle (ruler and compass) to construct figures, a modern interpretation of the problems has restricted the allowed solution methods to ruler and compass constructions, but none of the Greek attempts that have survived on any of these problems obey such a restriction, and, indeed, none of the problems can be solved by ruler and compass alone. Instead, solutions of various kinds were proposed, involving ingenious curves and novel construction methods, and there was considerable discussion about the validity of the methods that were used. A number of distinguished mathematicians joined in, Archimedes among them, and it seems that the problems focused attention markedly on significant challenges in mathematics.

In addition to the contributions to mathematics that the problems elicited, there is every sign that they caught the public's attention and were regarded as important. Socrates, in Plato's dialogue *Meno*, had drawn out

¹To speak of just classical problems is something of a misnomer. There were other equally important problems in classical times, such as the construction of a regular seven-sided polygon.

of a slave boy the knowledge of how to construct a square twice the size of a given square, thus demonstrating his theory of knowledge. Plato claimed that the analogous problem of duplicating the cube was ordained by the Gods, who required the altar at Delos to be doubled exactly. Less exaltedly, the problem of squaring the circle rapidly became a by-word for impossibility, and Aristophanes, a contemporary of Plato's, could get a laugh from an Athenian audience by introducing a character who claimed to have done it. Since all these problems possess simple, approximate, 'engineering' solutions, the Greek insistence on exact, mathematically correct, solutions is most striking.

To solve an outstanding problem is to win lasting recognition, as with the celebrated solution of the cubic equation by numerous Italian mathematicians at the start of the 16th century. In 1535, Tartaglia was challenged by one Antonio Fior to solve 30 problems involving a certain type of cubic equation. Fior had been taught the solution to the cubic by Scipione del Ferro of Bologna, who seems to have discovered it. As was the custom of the day, Tartaglia replied with 30 problems of his own on other topics, two months in advance of the contest date. With one day to go, Tartaglia discovered the solution method for Fior's cubics and won the contest and the prize, which was thirty dinners to be enjoyed by him and his friends. Such contests naturally promoted secrecy rather than open publication, because only the solutions but not the methods had to be revealed. Tartaglia later divulged the method in secret to Cardano, who some years later published it in his *Ars Magna* in 1545. Cardano argued that since the original discovery was not Tartaglia's, he had had no right ask that it be kept secret. Moreover, by then Cardano had extended the solution to all types of cubic equations, and his student, Ferrari, had gone on to solve the quartic equation as well.²

The tradition of setting challenging problems for one's fellow (or, perhaps, rival) mathematicians persisted. In 1697 the forceful Johann Bernoulli posed the brachistochrone problem, which asks for the curve joining two points along which a body will most quickly descend. He received three answers. Newton's he recognised at once: "I know the lion by his claw," he said. In fact, goaded by the way Bernoulli had wrapped the mathematical challenge up in the rapidly souring dispute over the discovery of the calculus, Newton had solved the problem overnight [40, p. 583].

Problems could be set to baffle rivals, but ultimately more credit resides with those who posed questions out of ignorance, guided by a shrewd sense of their importance. It is the lasting quality of the solution, a depth that brings out what was latent in the question, that is then recognised when the solver

²For some of the documents involved in this story, see [18, pp. 253–265]. Cubics were taken to be of different types because they were always taken with positive coefficients, so $x^3 + x = 6$ and $x^3 + 6 = x$ are of different types.

is remembered. Problems that point the way to significant achievements were systematically generated in the 18th century. This tradition was less successful in the 19th century, but was famously revived in a modified form by Hilbert in 1900. His choices of problems were often so inspired that those who solved one were said, by Hermann Weyl, to have entered the Honours Class of mathematicians [41]. It is this tradition of stimulating problems that the Clay Mathematics Institute has also sought to promote.

2. The Academic Prize Tradition in the 18th Century

The 18th century was the century of the learned academy, most notably those in Berlin, Paris, and St. Petersburg. To be called to one of these academies was the closest thing to a full-time research position available at the time, a chance to associate with other eminent and expert scholars, and the opportunity to pursue one's own interests. It was also a chance to influence the direction of research in a new and public way, by drawing attention to key problems and offering substantial rewards for solving them.

The academies ran their prize competitions along these lines. Problems would be set on specific topics. A fixed period of time, usually 18 months to two years, was allowed for their solution, a prize of either a medal or money was offered for their solution, and the solutions would usually be published in the academy's own journal. There was often a system of envelopes and mottos to assist anonymity, and success was liable to make one famous within the small world of the savants of the day. This was a group of some modest size, however, and was by no means confined to the very small group of mathematicians of the time. The historian Adolf Harnack (twin brother of the mathematician Axel) described the situation vividly in his history of the Berlin Academy of Sciences³:

In a time when the energies and the organization for large scientific undertakings — with the exception of those in astronomy — were still lacking, the prize competitions announced annually by the academies in Europe became objects for scientific rivalries and the criterion for the standing and acumen of scientific societies... This was so because specialties were most often disregarded and the themes chosen for competitions were either those that required perfect insight into the state of an entire discipline and its furtherance with respect to critical points, or those that posed a fundamental problem. The prize competitions constituted the lever by which the different sciences were raised one step higher

³This translation from [13, p. 12], original in [24, vol.1, pp. 396–397]. Reprinted with the permission of Cambridge University Press.

from one year to another; in addition, they were important for universalizing and unifying science. The questions were addressed to learned men all over Europe and were communicated throughout the scientific world. The suspense surrounding the announcement of the question was, in fact, larger than that of the answer, for it was in the formulation of the question that mastery was revealed. The invitation was not addressed to young recruits of science but to the leaders who eagerly answered the call to contest. The foremost thinkers and learned men — Euler, Lagrange, d’Alembert, Condorcet, Kant, Rousseau and Herder — all entered the arena. This circumstance which may seem quite strange today requires special explanation. This latter . . . resides in the fact that the learned man of the 18th century was still a *Universalphilosoph*. His mind could discern an abundance of problems in different scientific areas which all seemed equally attractive and enticing. Which one should he attack? At that moment, the Academy came to the rescue with its prize competitions. It presented him with a given theme and assured him a universally interested audience.

The first prize fund to be established was endowed by Count Jean Rouillé de Meslay, a wealthy lawyer, who left the Académie des Sciences in Paris 125,000 livres in his will in 1714 [13, p. 11].⁴ The Académie took this up,



Daniel Bernoulli

and from 1719 on, prizes were to be awarded every two years. The first two topics concerned the movement of planets and celestial bodies and, a related issue at the time, the determination of longitude. These were substantial issues. Newton’s novel theory of gravity, proclaimed in his *Principia*, was not widely accepted in Continental Europe. It sought to replace a clear physical process, vortices, with the much more problematic notion of action at a considerable distance, and it had a conspicuous flaw amid many striking successes: the motion of the moon. This particular failing was most unfortunate because the motion of the moon, if properly understood, could be a key to the longitude problem.

⁴The standard source of information is [32]. It should be pointed out that 125,000 livres was a very large sum of money; a skilled artisan of the period might hope to earn 300 livres a year.

Among the more famous winners of the Paris academy prizes was Daniel Bernoulli, who won no less than ten prizes, and most of his contributions show how important the topic of navigation was. His first success came in 1725, for an essay on the best shape of hour-glasses filled with sand or water, such as might serve as nautical clocks. In 1734 he shared the prize with his father Johann, who begrudged him his success, for an essay exploring the effect of a solar atmosphere on planetary orbits. Later successes included a paper on the theory of magnetism (joint with his brother Johann II) and on the determination of position at sea when the horizon is not visible. He also wrote on such matters as how to improve pendulum clocks.

The Academy of Sciences in St. Petersburg was established on the orders of the Emperor Peter the Great on January 28 (February 8), 1724, and was officially opened in December 1725, shortly after his death. To ensure that it worked to the highest standards of the time, Peter hired several leading mathematicians and scientists, Euler, Nicholas and Daniel Bernoulli, and Christian Goldbach among them. Euler was only 20 when he arrived, and he remained associated with the Academy for most of his life, publishing in its journal prolifically even when he was not an Academician.

In Berlin, the rival Academy of Sciences, the Académie Royale des Sciences et de Belles Lettres de Berlin, was founded in 1700, but it did not become influential until it was reorganised along Parisian lines in 1743 by Frederick the Great, who had come to power in 1740 and reigned until his death in 1786. He wished the academy to be useful to the state, and he paid the new staff he brought in high salaries, more than they would get in Paris but less than St. Petersburg. He installed Maupertuis as director of the academy, and Euler as director of the mathematical class. Maupertuis supported Voltaire's turn toward the English: Newtonian mechanics and Lockean metaphysics as opposed to Cartesianism. The first prize topic, for 1745, was 'On electricity' and was won by Waitz, the Finance Minister in Kassel. The prize amounted to some 50 ducats, and from 1747 took the form of a gold medallion. In 1746 d'Alembert won the prize for his essay 'Réflexions sur la cause générale des vents', which was his response to the challenge: 'Determine the order and the law which the wind must follow if the Earth was entirely surrounded on all sides by ocean, in such a way that the direction



Frederick the Great

and speed of the wind is determined at all times and for all places.’ Eleven entries had been submitted; d’Alembert’s is the first in which partial differential equations were put to general use in physics [39, p. 96]. The famous wave equation appeared in a paper d’Alembert published in the *Memoirs of the Berlin Academy* the next year, 1747.

As further evidence of the interest generated by the Berlin prize competition, Harnack noted that there were often a dozen entries for a given problem, although it was generally impossible to know who entered because only the names of the winner (and sometimes a runner-up) were ever announced. Young and old could enter, and could enter successive competitions; there was an explicit rule that in the event of a tie the foreign competitor was to be preferred. In the course of the 18th century, twenty-six different winners were German, ten French, two Swiss, and one each came from Italy and Transylvania.

There was naturally some overlap between the academies [24, p. 398]. Some of the same names occur in the lists of the other academies, and some more than once, the most notable case being that of Euler, who won no less than twelve prizes from various academies.



Leonhard Euler

All of this work entailed continual involvement behind the scenes judging the essays. Decisions were final, but were not always accepted gracefully: d’Alembert in the early 1750s complained that he was the victim of a cabal in Berlin that had denied him a prize for an essay on fluid mechanics (in fact, no one won the prize that year). He thereupon published his own essay, in 1752, in which he raised the paradox that the flow round an elliptical object should be the same fore and aft, which implied that there would be no resistance to the flow. It was left to others to find the flaw in d’Alembert’s argument, and meanwhile his relations with Euler worsened. The basic problem may have

been one of temperament. D’Alembert, although a charming conversationalist, was a slow writer who did not express his ideas with clarity. Euler was unfailingly lucid and wrote with ease. D’Alembert may have come to resent the way in which his ideas, once published, were so readily taken up and well developed by the other man. It was only in 1764, when d’Alembert tried actively to intervene with Frederick the Great on Euler’s behalf, that

relations between Euler and d'Alembert were put on a more amicable footing. D'Alembert's interventions were unsuccessful, however, and Euler left Berlin permanently for St. Petersburg in 1766.

Over the years a few problems recurred, mostly to do with astronomy and navigation. Euler won the Paris Academy prize of 1748 for an investigation of the three-body problem (in this case Jupiter, Saturn, and the sun). Then, knowing that Clairaut was wrestling with the inverse square law and was prepared to modify it, Euler proposed the motion of the moon as a prize topic for the St. Petersburg Academy in 1751.

Clairaut rose to the challenge, and suddenly found that he need not abandon Newton's law, as he had at first thought, but that a different analysis of the problem showed that the law could indeed give the right results. His successful solution to this problem was one of the reasons that the inverse square law of gravity became established and other theories died out. Other reasons included Clairaut's successful prediction of the return of Halley's comet in 1759. Comets are, of course, particularly sensitive to the perturbative effect of the larger planets, so the challenge of determining their orbits highlighted the importance of the many-body problem in celestial mechanics, which the Berlin Academy returned to again, for example in 1774.



Alexis Claude Clairaut

The Paris Academy in 1764 asked for essays on the libration of the moon: Why does it always present more or less the same face to us, and what is the nature of its small oscillations? In 1765 they asked about the motion of the satellites of Jupiter, and the competition was won by Lagrange (who was then 29).

Both these topics reflect the hope that celestial motions could somehow be interpreted as clocks and so solve the longitude problem. In 1770 the prize went jointly to Euler and his son Albrecht for an essay on the three-body problem, and in 1772 the same topic again led to the prize being shared, this time between Euler and Lagrange. In 1774, Lagrange won again, for an essay on the secular motion of the moon, but he had begun to tire of the subject and needed an extension to the closing date, which d'Alembert requested Condorcet to offer as an inducement to continue. Lagrange refused to enter the next competition, on the motion of comets — the prize went to Nicholas

Fuss — but he entered the competition on the same topic in 1780 and won the double prize of 4,000 livres. Thereafter he never entered a competition of the Paris Academy [26].

Prizes could be set to address embarrassing deficiencies in the state of the art. Lagrange, a member of the Berlin Academy since 1766, persuaded it to ask for a rigorous foundation of the calculus in 1784. The prize was to be awarded for ‘a clear and precise theory of what is called Infinity in



Joseph Louis Lagrange

mathematics’. The continual use of infinitely large and infinitely small quantities in higher mathematics, noted the preamble, was successful despite seeming to involve contradictions. What was needed was a new principle that would not be too difficult or tedious and should be presented ‘with all possible rigour, clarity, and simplicity’ [20, pp. 41–42]. The tedious approach the academy wished to head off was the defence of the Newtonian calculus that MacLaurin had mounted, which replaced Newton’s intuitive limiting arguments with the cumbersome apparatus of double reduction ad absurdum.

The competition was won by Simon L’Huillier, and two essays written for it made their way into books (L’Huillier’s [27] and Lazare Carnot’s [8]). The judges were satisfied with neither, however, and, when the newly founded École Polytechnique required Lagrange to publish his lectures he produced, his own account, the *Fonctions analytiques* of 1797. This entirely algebraic account lasted until Cauchy’s analysis began to sweep it away in the 1820s.⁵

3. The Academic Prize Tradition in the 19th Century

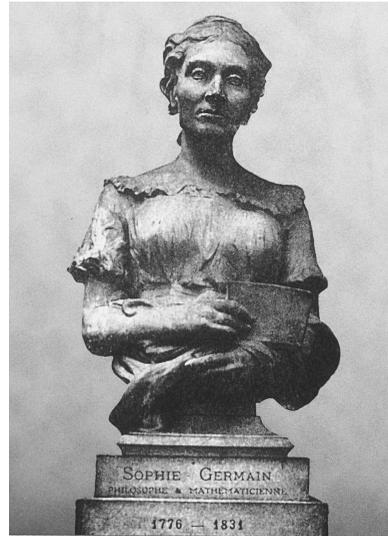
After the French Revolution, the revised Académie in Paris had two new prizes, starting in 1803, of 3,000 FFr: the grand prix des sciences mathématiques or the Grand Prix des sciences physiques. The two were intended to alternate, and a professor’s salary at the time was some 4,000 FFr, so the prize was indeed generous. There were also some irregular prizes, such as the competition proposed by Napoleon in 1809 on the vibrational modes of elastic plates. This was in response to Chladni, who had come to Paris and demonstrated many new experiments on this unstudied phenomenon. Laplace was in charge of the commission that was to judge the prize,

⁵See [9]. The facsimile re-edition edited by U. Bottazzini has a very useful introduction.

and he hoped that it would provide an occasion to advance his protégé, Poisson. The prize competition was officially announced for 1811 and drew only one entry, not, however, from Poisson, but one written by the unknown Sophie Germain.⁶ The judges found it inadequate, and the competition was extended to October 1813. Germain worked to deepen her analysis, and hers was again the only entry. She was by now in correspondence with Legendre, who was one of the judges, and he seems to have been disappointed with her work, although it now obtained an honourable mention. The competition was extended again, to October 1815. Only now did Poisson submit a memoir, but since he had been a judge of this very competition in 1813 his actions were irregular to say the least, and Legendre protested. The memoir was nonetheless read to the Institut de France and a note about it inserted in the *Correspondance de l'École Polytechnique*, where it was said that it might prove helpful to potential competitors. Poisson seems to have hoped that by acting in this way the question would be permanently withdrawn while he nonetheless earned the approval of Laplace. However, his actions were so scandalous that a deal seems to have been struck to keep the question open, and possibly even to give a prize to Sophie Germain if she could improve her work sufficiently. This in the end, she did, not mathematically, but experimentally, and she was awarded the prize in 1815.

As Germain's story shows, the administration of prizes in the small hot-house environment of Paris was not without problems. It may have discouraged Germain from entering the competition on Fermat's Last Theorem, which was the topic set in 1815 for 1817. In fact, no one entered, and after four years the question was withdrawn, but in that time Germain had made one of the few notable inroads on the topic in the century between Euler and Kummer [17, p. 62]. These results made her famous when Legendre published them as hers in his *Théorie des Nombres* in 1830.

Further evidence of the way the prize competition worked at the start of the 19th century is provided by the mathematics prize for 1815. This was won by Cauchy who, in answer to a question about the propagation of waves, wrote a memoir chiefly remarkable for his discovery of the way a



Sophie Germain

⁶This account follows that in [6].

function and its Fourier transform are inter-related, made in ignorance of Fourier's own work. His memoir was not published, however, until 1827, by which time it had long been eclipsed by Poisson's independent discovery and prompt publication of many of the same results [4, p. 90].

A notable success of these prizes came in 1819, when the mathematics prize was won by Fresnel in a decisive moment for the wave theory of light. But hints of what was to come are visible in another celebrated award of the prize, which went jointly to Abel (posthumously) and Jacobi in 1830 for the independent discovery of elliptic functions, not because the topic had been identified in advance but because their work was rapidly recognised after the event to be a momentous discovery (and because Legendre was in a position to see that the Académie could offer such a prize). During the 19th century, the original tradition of what might be called prospective prizes (titles announced in advance and a specific deadline to be met) came under pressure, and the alternative of retrospective prizes or general subject area prizes for work in some area of mathematics or science were proposed. This was more and more the case as new prizes were established, but even when the title was precise the judges began to allow previous work to be submitted, rather than rolling the topic over for two more years (which was also done).

The difficulties that arose when a question attracted no good entries were considerable. It was embarrassing, and there were financial implications. The first hint of an alternative solution that preserved outward appearances had come in 1810, when Lagrange and Laplace jointly proposed the double refraction of light as the Grand Prix topic for 1810, knowing very well that the 35-year-old Etienne Malus was at that moment doing brilliant work in optics. He did indeed win, and happily the challenge inspired him to extend his earlier work considerably; during the process he discovered the polarisation of light [14, pp. 271–272]. This same thing happened in 1812, when heat diffusion was the topic, upon which Fourier was known to be at work, and this time the result was that great rarity, a work as important in the history of physics as it is in the history of mathematics.

As the 19th century wore on, the Grand Prix in mathematics had mixed results. A question on the perturbations of elliptic orbits was first set in 1840 but only answered successfully (by Hansen) in 1846, but other questions, on the maxima and minima of multiple integrals and on Abelian functions, were answered successfully within the initial two-year period, by Sarrus and Rosenhain respectively. Then the commissions ran out of luck for a while. Fermat's Last Theorem was proposed in 1850, with Cauchy as the chairman of the judges, but no satisfactory answers were received, and the problem was rolled over to 1853 before being abandoned. The spur for this was Lamé's argument using cyclotomic numbers, in 1847, in which he mistakenly

supposed that such integers have a unique factorisation law. His error was pointed out by Liouville, but this only inspired Cauchy to claim that he could solve the problem, and order was not restored until Liouville brought Kummer's much more profound ideas to France [17, 30]. The distribution of heat in an infinite body was the topic proposed in 1858 and finally withdrawn in 1868. This competition is remembered only because Riemann's entry was passed over — the jury found that the way in which the results had been discovered was insufficiently clear.

In 1865 Bertrand was in charge of a question asking for an improvement to the theory of second-order partial differential equations, but there were no answers, and the question was repeated; it was answered to the satisfaction of the panel by Bour in 1867. There were no answers to the question Bonnet set in 1867 on algebraic surfaces, and none to Puiseux's question (the three-body problem) in 1872. Singular moduli and complex multiplication in the theory of elliptic functions drew no response in 1874, nor did the suggestion that elliptic and Abelian functions might be profitably applied to the theory of algebraic curves in 1878.

However, in between those years Darboux did win the prize for an essay on singular solutions of first-order partial differential equations.

In 1880 the Grand Prix was again awarded, for an essay 'significantly improving the theory of linear ordinary differential equations'. The prize went to G.H. Halphen, for an essay on the invariants associated to a differential equation, but the competition is best remembered for the second-place entry from Poincaré on the theory of automorphic functions and the relationship between non-Euclidean geometry and the nascent theory of Riemann surfaces (see [35] or [22] for fuller accounts).

In 1882 embarrassment came to the Paris academy. With Camille Jordan in charge, they proposed an investigation of the number of ways a number can be written as the sum of five squares. The young German mathematician Hermann Minkowski, then only 18, and the English mathematician H.J.S. Smith submitted entries that shared the prize. Unfortunately, Smith's contribution was confined to showing that he had already solved the problem some years before. To make matters worse, by the time the result was announced, Smith had died. Hostile critics pounced on this to suggest that Minkowski must have known of Smith's work because he was surely too



Bernhard Riemann

young to have done the work on his own, and the ensuing row carried ugly hints of anti-semitism before the academy rightly pronounced itself satisfied that Minkowski had been entirely independent of Smith [15].

They had better luck in 1886, when Halphen proposed a question generalising the regular solids that Goursat answered; in 1888 when Poincaré asked about algebraic equations in two independent variables, and Picard was awarded the prize; and in 1894 when Darboux asked for an improvement in the theory of the deformation of surfaces, and the commission was able to award the prize to Weingarten. A more famous award came in 1892 when Hermite persuaded the academy to ask for the determination of the number of prime numbers less than a given number (the prime number theorem), the aim being to draw mathematicians to fill some of the gaps in Riemann's famous paper of 1857. He hoped in this way to get his friend Stieltjes to write up the details in support of his 1885 claim to have solved the Riemann hypothesis. As the closing date drew near, and even though Hermite wrote to Stieltjes to encourage him, no essay was forthcoming. Instead, the young Hadamard presented his doctoral thesis on entire functions in 1890, and Hermite, who was one of the examiners, suggested that Hadamard find applications for his ideas. Hadamard confessed that he had none, but he soon realised that his new theory was just what was needed to resolve the prime number theorem, and he submitted a long essay to that effect, which was awarded the prize on 19 December 1892 [33, pp. 55–57]. Stieltjes never found the proof he had incautiously claimed.

These competitions continued after World War I, when Julia, Lattès, and Pincherle wrote essays on iteration theory [1, pp. 108–116]. The prize went to Julia, with an honourable mention and 2,000 FFr to Lattès. Fatou, who had decided not to enter the competition, was also awarded 2,000 FFr. His work and Julia's were strikingly similar, and at Julia's request the question of independence and priority was addressed directly. It was found that Julia's results, presented in a sealed letter to the academy as the competition rules required, did indeed predate Fatou's publications, but that the men had worked independently.

Regardless of the problems administering the prizes, donors found them attractive, until by 1850 there were thirteen different French prizes across the sciences, all controlled from Paris. The number rose again after the defeat of France in the Franco-Prussian War (1870–71), when there was a widespread feeling that science had been allowed to decline too far and thus contributed to the national defeat. The new prizes, like the more established ones, were overwhelmingly retrospective, but to make them more attractive it was argued that the reward should be financial rather than in the form of a medal. The impact of these prizes was considerable, amounting to one third to half of the winner's annual salary, depending on whether or not he or she

lived in Paris, and was often a huge boost to scientists when equipment was needed.

The strictly mathematical prizes participated in this general shift. The prix Poncelet was endowed by the wife of General Jean-Victor Poncelet after his death in 1868 in order to carry out his dying wish that the sciences be advanced. Poncelet himself had been one of the chief creators of projective geometry in the 1820s, before turning to the theory of machines. His widow's generosity was augmented by a further sum of money, and the prize of 2,000 francs was inaugurated in 1876. It operated invariably as a retrospective prize.

The prix Bordin was created by the will of Charles-Lauren Bordin, who died in 1835 leaving the Institut de France 12,000 francs. The institute eventually created an annual prize of 3,000 francs after the Company of Notaries had declined a similar bequest, and the first prize was awarded in 1854. Topics moved from theoretical physics to pure mathematics. In 1888, for example, the prize was offered for an essay improving in some important way the theory of motion of a solid body. There is good evidence [11] that the topic was set with Sonya Kovalevskaya's work in mind.



Sofia Kovalevskaya

Kovalevskaya wrote to Mittag-Leffler over the summer of 1888 to say that “Bertrand gave a large dinner in my honor, attended by Hermite, Picard, Halphen, and Darboux. Three toasts were proposed in my honor, and Hermite and Darboux said openly that they have no doubt that I shall have the prize” [11, p. 114]. Since Bertrand was the perpetual secretary of the academy, Hermite was the most influential French mathematician behind the scenes, and Darboux was on the panel of judges, they presumably knew whereof they spoke. They had already extended the closing date so that her essay could be received — it arrived late in the summer — and the prize was awarded to her in December. That said, she won the essay for a fine piece of analysis applying the theory of Abelian functions to rigid body motion, thus showing how the new functions had their uses in physical problems.

In 1892 the advertised topic was in differential geometry, and Gabriel Koenigs won with an essay on geodesics. In 1892 the topic was the application of the theory of Abelian functions to geometry, then the domain of

a rising star, George Humbert, who duly won. In 1894 Paul Painlevé and Roger Liouville shared the prize for their essays on the use of algebraic integrals in problems in mechanics. In 1896 Hadamard won with an essay on geodesics on surfaces, one of the few to respond to the work of Poincaré on the same subject. The feeling that all these topics were set with a shrewd eye to who was working actively on what subject deepens with what became one of the more awkward incidents in the history of the prize [10, pp. 58–59]. The first draft of the paper Enriques and Severi submitted to the committee of the Bordin prize was flawed. They became aware of these mistakes after a conversation with de Franchis, and withdrew their paper only to make some corrections and to re-submit it, even though they knew that Bagnera and de Franchis were also candidates for the same prize, and indeed had better results. The prize went to Enriques and Severi, and de Franchis complained through the intermediary of Guccia, the well-respected editor of the Palermo *Rendiconti* (Bagnera and de Franchis were also Sicilian). As a result, the same topic was advertised again as the prize for 1909, and this time Bagnera and de Franchis won the prize.

The early years of the Steiner prize from the University of Berlin illustrates the problems of prize competitions only too well. It was endowed in the will of the distinguished exponent of synthetic geometry, the Swiss mathematician Jacob Steiner, who had taught most of his life at Berlin University and died in 1863. Steiner stipulated that the prize, of 8,000 Thaler, be awarded once every two years for a geometric topic treated synthetically.⁷ The first time the prize was awarded, 1864, it was divided between Cremona and Sturm for their answers to a question set by Weierstrass concerning cubic surfaces, a currently active topic. In 1868 the prize was shared between Kortum and H.J.S. Smith for works on cubic and quartic curves in the plane. The competition ran easily enough as long as Steiner's followers were still alive, but soon successors proved hard to find. In 1870 Borchardt proposed the topic of lines of curvature on surfaces, but no essays were forthcoming, and in 1872 the prize went to Hesse for his work in geometry as a whole, and in 1874 to Cremona, again in recognition of his work in general. In 1874 the judges called for entries on the theory of polyhedra, but none were forthcoming and the topic was withdrawn in 1876. Instead, the prize was awarded to H. Schröter for his work extending and deepening Steiner's geometrical methods. The judges then announced a prize of 1,800 marks for an essay on higher algebraic space curves, but there were no entries. The closing date was extended to 1880, and the prize was awarded to Theodor Reye 'for his distinguished work on pure geometry'. In 1880 there was still no satisfactory entry. The judges awarded the prize to L. Lindelöf for a

⁷In 1871 the Thaler was replaced by the German mark, at a rate of 1 Thaler = 3 German marks = 75 U.S. cents. The prize was worth well more than the average annual income of a teacher.

solution to Steiner's problem about the maximum volume of polyhedra of a given type and further extended the closing date for the essay originally set in 1876. Finally, in 1882, they announced two significant essays that were worthy of sharing the prize: Max Noether's and Henri Halphen's. A third essay received an honourable mention, but the author's name was not revealed (most likely it was Rudolf Sturm, who promptly published on that topic). And so it went on. In 1884 and 1888 Fiedler and Zeuthen were rewarded for their distinguished contributions to geometry. Only in 1886 was the prize awarded, to Ernst Kötter, for an essay on the question proposed in 1882 and modified in 1884, which called for a theory of higher curves and surfaces that invoked really existing objects to replace the imaginary points, lines, and planes of contemporary algebraic geometry.

In 1888 Kronecker, with the support of Fuchs, asked that the terms of reference of the prize be changed. This was difficult to achieve, but in late 1889 it was agreed that from 1890 the competition would be announced once every five years, and in the event that no entry was judged satisfactory the prize money could be allocated to significant work, primarily in the field of geometry, written in the previous ten years — a marked relaxation of the original rules. In this spirit, no entries having been received on the set topic (lines of curvature on surfaces, again) Gundelfinger and Schottky shared the prize in 1895. In 1900 Hilbert was awarded a one-third part of the Steiner prize for his work on the foundations of geometry (*Grundlagen der Geometrie*, 1899).

The other winners were Hauck and Lindemann. It was the same story in 1905, when the prize went to Darboux. One is forced to conclude that not even a prize could rescue the methods of synthetic geometry from entering into a prolonged eclipse.

Harnack noted that prize competitions were no longer favoured by the Berlin Academy and declined in importance, and on a number of occasions the prize had to be held back for want of a good enough entry [24, p. 397]. In fact, the prize competition organised by the Berlin Academy had terrible results in the 19th century. It got off to an unfortunate start in 1836 when a question set by Dirichlet asking for numerical methods for solving polynomial equations with real or complex roots drew no answers. In 1840 Dirichlet replaced this question with another, inspired by the recent work of Abel, about integrals of algebraic functions. This question also drew no response by the closing date in 1844, and he replaced it with a third, in 1852, where he asked for a proof that the differential equations of dynamics cannot in general be reduced to integrals but require the introduction of new analytic expressions. Dirichlet had his friend Jacobi's work in mind, as he had done in setting the earlier problem, in this case Jacobi's analysis of the spinning top. Yet again there were no answers. In 1858, by which

time Dirichlet had moved to Göttingen, Borchardt took up the challenge of setting an attractive question. He proposed the subject of lines of curvature on surfaces, to no avail: Only one, unsatisfactory, essay was received. Finally, in 1864 Weierstrass proposed the topic of finding a significant problem whose solution requires the new elliptic or Abelian functions, and the academy was able to award the prize for the first time, to Weierstrass's former student Schwarz for his work on minimal surfaces.

Repeated failure must have given the academicians pause, because the prize was not offered again until 1894, when Lazarus Fuchs offered a topic arising out of his own work on differential equations. This attracted no entries, and was re-advertised, in a slightly altered form, in 1894, again without success. In 1902 the prize was awarded to Mertens, a former graduate of Berlin by then in Vienna, for his contributions to mathematics, and Fuchs's question was re-advertised as a question about functions of several variables which are invariant under certain linear transformations. There were no entries, and in 1910 the prize went to Koebe for his work on the uniformisation theorem, and Frobenius asked a question about the class number of the most general cyclotomic field. By 1914 there were no entries, and World War I brought this dismal sequence to an end.

Other societies were more carefully managed. The Jablonowski Society (more properly, the Fürstlich Jablonowshi'schen Gesellschaft der Wissenschaften) was founded in Leipzig by the Polish Prince Jablonowski in 1774 after some years as a private institution. He used it to propose prize problems and sponsored a journal. Problems were set in the domains of mathematics and physics, economics, Polish history, and the history of the Slavs. The society's members were the professors of the University of Leipzig, and they were responsible for running the competitions. In the early years of the 19th century the prizes became unattractive, but the society's finances prospered, and in 1846 it was influential in founding the Königlich Sächsischen Gesellschaft der Wissenschaften zu Leipzig. They benefited from the affair of the Göttingen Seven — seven professors, Gauss's friend and colleague the physicist Wilhelm Weber among them, who were expelled from the university of Göttingen for refusing to accept the terms imposed by the Duke of Cumberland. Weber moved to Leipzig and was one of a number of scientists who built up the university's reputation considerably in the 19th century. The society remained active until 1948, when its leader moved to Jena, and was refounded with help from the Polish government in 1978 with the aim of improving German–Polish economic and cultural relations.

Guided by the Leipzig professors, the Jablonowski Society had more success than many in proposing suitable topics. The first time the prize was awarded after the mid-century reforms, in 1847, it went to H. Grassmann

for work connecting geometric analysis to Leibniz's geometric characteristic. In 1884 the society called for essays on the general surface of order 4, extending the work of Schläfli, Klein, Zeuthen and Rodenburg on cubic surfaces, and gave the prize two years later to Karl Rohn. In 1890 they asked for essays extending the work of Sophus Lie on the invariant theory of arbitrary differential equations, and in 1893 they received an essay from M.A. Tresse that he completed in 1895; he was awarded the prize in 1896. Tresse was a student of Engel's and Lie's in Leipzig. In 1902 they asked for an essay which would essentially complete the work of Poincaré on Neumann's method and the Dirichlet problem. This was a propitious theme, and prizes went to E.R. Neumann in 1905, Plemelj in 1911, Neumann again in 1912, and Gustav Herlotz in 1914.

The Danish Academy of Sciences also awarded prizes from time to time during the 19th century. These seem to have been managed in a traditional way, with titles announced in advance, and in 1823 they had notable success when they awarded Gauss the prize for his essay on the conformal representation of one surface on another. Other prizes were awarded in 1875, to Schubert for his work on the enumerative geometry of cubic curves in space, and to Gram in 1884 for a paper on the prime number theorem. Still, in these cases one notices that the commission charged with conducting the prize had a shrewd eye to success. In the 1820s Schumacher, a prominent geodesist and astronomer, who had organised a survey of Denmark, knew very well that Gauss was conducting a detailed survey of Hanover that was intended to extend the Danish survey, because he was in extensive correspondence with him. What is more, the prize question was formulated by Gauss himself, who then abstained from entering the competition himself for two years before submitting his essay [7, p. 102 n.]. In the 1870s the Danish geometer Zeuthen was particularly attracted to enumerative geometry, a subject in which Schubert was emerging as the leading figure. Prizes otherwise seem to have been set almost every year, more often on applied than on pure topics, and generally with little success (only 14 times in the years 1800–1886 for which records are easily accessible).⁸

A rare example of a successful prize competition is the one organised by the Swedish mathematician Mittag-Leffler at the request of the Swedish



Jules Henri Poincaré

⁸I thank Jesper Lützen for this information and his guidance in my reading of the standard source [29].

king Oscar II, who wished in this way to mark his 60th birthday, but even this illustrates the perils and pitfalls of such competitions [3]. Mittag-Leffler enlisted Weierstrass and Hermite as judges, and together they proposed four topics, while reserving the right to award the prize to any valuable entry on the theory of functions if none of the questions were adequately answered.



King Oscar II of Sweden

Their choice of questions provoked Kronecker to claim that the fourth of these had already been proposed, and indeed that he had solved it, but this allegation eventually petered out. By the closing date twelve entries had been submitted (there was also a late entry from an English angle-trisector). One entry stood out, Henri Poincaré's on the three-body problem. Poincaré had not only submitted a memoir, but he had added to it as time went by in answer to questions about his work that Mittag-Leffler had sent to him. This outraged Weierstrass, who insisted that such irregular behaviour would never have been contemplated in Berlin. In due course Poincaré was declared the winner, with Appell an honourable second.

Poincaré was awarded the prize of 2,500 kroner and a gold medal, and the printers of *Acta Mathematica* were instructed to start printing his revised manuscript.⁹ Even at this stage Mittag-Leffler's editorial assistant Phragmén was raising points in the memoir that he did not understand, and in answer to one of these Poincaré admitted that the memoir as it stood was in serious error. Mittag-Leffler ordered the printing halted and all copies of the printed version destroyed, doubtless to prevent his hostile critics on the editorial board of *Acta* and beyond from finding ammunition in the debacle. Then Poincaré found a way to profit from the mistake, and in so doing created the theory of what are now called homoclinic tangles thus opening the way to a mathematical theory of chaos. Mittag-Leffler was happy to print the revised manuscript, but he also charged Poincaré the full printing costs, which came to more than the original prize money. So although the prize competition called forth several good entries and one major paper in the history of mathematics, the stresses involved were too great. No further royal competitions in mathematics were organised, and the king, who had studied mathematics as a young man, found other ways

⁹Barrow-Green [3, p. 58 n. 101] citing [16] observes that Mittag-Leffler's annual salary at the time was 7,000 kroner.

to support mathematicians. For example, he rewarded Kovalevskaya, who was by then a professor in Stockholm, for a paper extending the work for which she had won the prix Bordin.

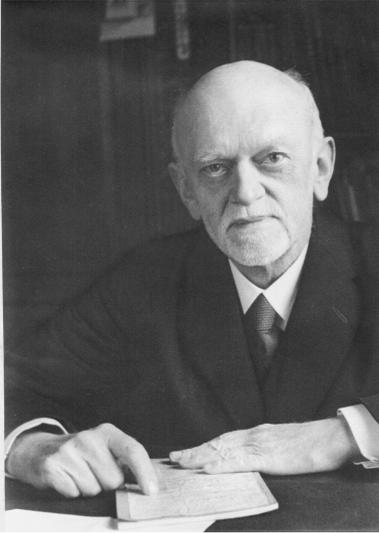
Another well-known prize for the solution of a specific problem is the recently awarded Wolfskehl prize, offered for a solution of Fermat's Last Theorem.¹⁰ This prize was first established by Paul Wolfskehl, who came from a wealthy, charitable Jewish banking family. Paul Wolfskehl trained as a doctor but took up the study of mathematics when multiple sclerosis made it clear to him that he would soon be unable to practice. It is very likely that the solace he found in mathematics during the long years of his illness inspired him to create the prize. Wolfskehl died on 13 September 1906, and according to the terms of his will, 100,000 gold marks were set aside for the correct solution of the problem. The Royal Society of Science in Göttingen was charged with administering the fund and adjudicating the solutions. Conditions for the prize were settled and published in 1908, and there was a closing date of almost a century hence: 13 September 2007. A proof of Fermat's Last Theorem, or, if it is false, a characterisation of the exponents for which it is true, would qualify for the prize, but a mere counterexample would not.

From some perspectives, such as generating enthusiasm for mathematics, the prize was a great success; from others, such as the advancement of knowledge, it was a complete disaster. In the first year no fewer than 621 solutions were submitted, and over the years more than 5,000 came in. These had to be read, the errors spotted, and the authors informed, who often replied with attempts to fix their 'proofs'. One can only assume that most, if not all, of the authors knew very much less than Wolfskehl himself about the depth of the problem, but one of them was Ferdinand Lindemann (famous for his proof that π is transcendental) who failed twice, in 1901 and 1908. Much work was also done in Berlin handling correspondence about the prize. Here another doctor with a love for mathematics, Albert Fleck, dealt with so many attempts on behalf of the Berlin Academy of Sciences, Lindemann's among them, that he was eventually awarded the Society's silver Leibniz medal in 1915 for his work; mathematicians in Berlin referred to his operation as the 'Fermat Clinic'. Estimates have varied over the years, too, about the cash value of the prize. It turns out to have been prudently invested and to have survived the strains of high inflation and the vicissitudes of German history better than was often said. Had it been kept safely in gold, its present value would be around \$1,400,000. When finally it was awarded to Andrew Wiles in 1997, it was worth DM 75,000 (approximately \$37,000) but many estimates had by then written it off and the best put it at around DM 10,000.

¹⁰This account follows [2].

4. The Hilbert Problems

The most successful attempt to reverse the trend toward retrospective prizes and to set problems on topics that would actually bring forth new work is, of course, that of David Hilbert, who proposed 23 problems at the International Congress of Mathematicians in Paris in 1900.¹¹ His thrilling opening words captured exactly the appeal of great problems: “Who among us,” began Hilbert, “would not be glad to lift the veil behind which the future lies hidden; to cast a glance at the next advances of our science and at the secrets of its development during future centuries?” His close friend Minkowski had encouraged him to seize the opportunity to shape the next century in mathematics, writing to him that “Most alluring would be the attempt to look into the future, in other words, a characterisation of the problems to which the mathematicians should turn in the future. With this, you might conceivably have people talking about your speech even decades from now. Of course, prophecy is indeed a difficult thing” [36, p. 119].



David Hilbert

The actual speech on the day was something of a disappointment, but in their written, published form the problems gradually worked their charm on the mathematical community. The text is infused with Hilbert’s confidence that any problem can be solved. He liked to proclaim on various occasions that there is no ‘ignorabimus (we shall not know) in mathematics’. He regarded great problems as crucial for the growth of mathematical knowledge, and he took two as exemplary: Johann Bernoulli’s brachistochrone problem, and Fermat’s Last Theorem. The first is rooted in empirical sources, the second in the purely mental thought processes of human beings, and creative mathematics, for Hilbert, moves

between the two. Thus problem solving and theory formation go hand in hand. The brachistochrone problem had initiated the calculus of variations, a branch of mathematics about to absorb some of Hilbert’s own attention. Fermat’s Last Theorem had already led to Kummer’s work on ideal numbers and thence to the theory of algebraic number fields, the subject of Hilbert’s *Zahlbericht* of 1897.

Between these two sources, the applied one augmented by mention of Poincaré’s recent solution of the three-body problem, Hilbert placed a num-

¹¹See [23, 41] and numerous studies of the individual problems.

ber of contemporary developments demonstrating the unity of mathematics as he saw it. Whereas Poincaré on such occasions always sought to emphasise the importance of applications, Hilbert asserted that it was problems rooted in purely mental thought processes that gave rise to practically ‘all the finer problems of modern number and function theory.’ The result was the miraculous pre-established harmony that the ‘mathematician encounters so often in the questions, methods, and ideas of various fields;’ Hilbert gave the example of Felix Klein’s study of the Platonic solids, which wove a complex theory that connected geometry, group theory, Riemann surfaces, and Galois theory with the theory of linear differential equations.

The specific problems Hilbert raised, and there are more than 23 because several problems come in families, are of various kinds, and cannot all be considered here [23, 41]. Some are more like programmes, of which the sixth is the most ambitious. It called for an axiomatisation of physics: Hilbert had recently axiomatised geometry, which he saw as the best-understood branch of science. He imagined that mechanics was ripe for similar treatment and hoped that each branch of science could be dealt with in the same way, because he felt that only an axiomatised theory could respond well to the discoveries made by the experimenters. Indeed, the first six problems form a coherent group focused on foundational questions. The first is the continuum hypothesis, identified by Hilbert as the most interesting problem of the day in set theory. The second calls for foundations for arithmetic, necessary because Hilbert’s axiomatisation of geometry rested on otherwise undefended assumptions about number. The first of these was already a significant problem in mathematics, but the second was original with Hilbert and proved difficult to sell, partly because the Italians, who were strong in this area, bridled at Hilbert’s intervention, and partly because, at that time, Hilbert had very little idea how it might be solved.

The next six problems belong to number theory and are largely algebraic: the transcendence of certain numbers, the Riemann hypothesis and the distribution of prime numbers, and the solvability of any Diophantine equation may be mentioned here. It is noteworthy that Hilbert’s hunches were often wrong. The problem he thought might go first, the Riemann hypothesis, is still with us, but the transcendence questions were solved relatively soon, in the 1930s. The next six are largely geometric and of a more specialist appeal, while the last five are in analysis, the direction in which Hilbert’s own interests were going. To cite just three of these, he proposed that a class of what he called ‘regular’ problems in the calculus of variations should have analytic solutions, he asked for a study of boundary value problems and the Dirichlet problem, and he asked about the general theory of calculus of variations.

The selection of problems was remarkably well done. It is possible, and interesting, to note that entire topics are missing that soon became major areas of 20th century mathematics (topology, measure theory and the Lebesgue integral, for example), but it is more important to note that, in contrast to every learned society in the previous hundred years, Hilbert picked topics that people wanted to work on. Hilbert's problems did not have a closing date of two years hence and a panel of judges appointed to evaluate them; had that been the case there would indeed have been little to show in the first few years. But Hilbert was aiming for longer-term success, and this he achieved. The designated problems are an astute mixture of those known to be important and those deriving from his own experience as a mathematician, which was already broad and was rooted in the fertile soil of the university of Göttingen. Very few seem unduly narrow, and several have been profitably reformulated in light of later experience, a sure sign that Hilbert was on to something deep.

Hilbert shrewdly allowed that a proof that something could not be done counted as a solution, and not as an indication that there are things in mathematics we shall not know. He was also fortunate that many of his problems retained interest even when it became clear that they were not going to have the solution he expected. Solutions, as Hilbert astutely recognised, could also be in the negative, as long as a genuine proof was given that the answer could not be found by the stated means. His paradigm example was the work of Abel and Galois that showed that the quintic equation could not be solved by radicals.

Hilbert spoke of problems in mathematics in terms that can be echoed today. Problems were a sure sign of life. The clarity and ease of comprehension often insisted upon for a mathematical theory, "I should still more demand for a mathematical problem if it is to be perfect; for what is clear and easily comprehended attracts, the complicated repels us. Moreover a mathematical problem should be difficult in order to entice us, yet not completely inaccessible, lest it mock our efforts. It should be to us a signpost on the tortuous paths to hidden truths, ultimately rewarding us by the pleasure in the successful solution." He argued in favour of rigour on the grounds of simplicity, and extended this requirement as far as geometry, mechanics, and even physics. He suggested that both generalisation and specialisation had valuable roles to play in tackling problems.

5. Some Famous Retrospective Prizes

In 1895 the University of Kasan established a prize to recognise the achievements of their distinguished rector in the early days of the university, Nicolai Ivanovich Lobachevskii, one of the discoverers of non-Euclidean geometry. The Lobachevskii prize was to be awarded for the best recent

book on a geometrical subject, and particularly for books on non-Euclidean geometry. It was awarded for the first time in 1897, when it went to Sophus Lie for the third volume of his *Theorie der Transformationsgruppen* [1893]. Klein proposed Lie for the prize, a magnanimous gesture on his part since Lie made what verged on a personal attack on Klein in the preface to that book. The second time the prize was awarded was in 1900, when it went to Wilhelm Killing for the second volume of his *Einführung in der Grundlagen der Geometrie*. In 1904 the prize went to David Hilbert, for whom Poincaré wrote a very strong recommendation adapted from his highly positive review of Hilbert's *Grundlagen der Geometrie*. In this connection it is amusing to note that in 1905 the first award of the Wolfgang Bolyai prize of the Hungarian Academy of Sciences went to Poincaré, while Hilbert received a special citation, and in 1910 the second Bolyai award went to Hilbert with Poincaré again the author of a glowing tribute. Sadly, this prize lapsed during World War I¹² but the Lobachevskii prize continues almost uninterrupted to this day and numbers among its most distinguished recipients Hermann Weyl (1927) and Kolmogorov (1986).

The Nobel prizes were established in the will of Alfred Nobel (1833–1895), in which he created a fund: “the interest on which shall be annually distributed in the form of prizes to those who, during the preceding year, shall have conferred the greatest benefit on mankind.” These benefits were to be found in work on physics, chemistry, physiology or medicine, literature, and peace. There is no reason to suppose that Nobel seriously contemplated a Nobel prize in mathematics, which was not and is not self-evidently beneficial to mankind in the way the designated topics are.¹³ Nonetheless, Mittag-Leffler does seem to have hoped that Nobel might have donated money to the Swedish Hogskola (the precursor of the University of Stockholm) and to have begun negotiations with him with that aim in mind. He was very disappointed when Nobel did not do so. As Crawford writes: “although it is not known how those in responsible positions at the Hogskola came to believe that a large bequest was forthcoming, this indeed was the expectation, and the disappointment was keen when it was announced early in 1897 that the Hogskola had been left out of Nobel’s final will in 1895. Recriminations followed, with both Pettersson and Arrhenius [academic rivals of Mittag-Leffler in the administration of the Hogskola] letting it be known that Nobel’s dislike for Mittag-Leffler had brought about what Pettersson termed the ‘Nobel Flop’ ” [13, p. 53]. In any case, the Nobel prizes, in line with 19th century experience, were entirely retrospective in nature.

¹²It was revived in 2000, when the prize went to S. Shelah for his *Cardinal Arithmetic*, Oxford University Press, 1994.

¹³Economics was added in 1968, and one may wonder, once the dismal science has been admitted, what else might one day qualify.

During the second half of the 20th century the Fields Medals, awarded every four years at the International Congress of Mathematicians (ICM), established themselves as the most prestigious prize in mathematics. They were established in the will of the Canadian mathematician John Charles Fields (1863–1932). He had been involved in the organisation of the ICM in Toronto in 1924, from which German mathematicians were excluded because passions were still running intensely after World War I. This pained Fields, who had been educated in Germany, so he endowed the medals, which were awarded for the first time at the ICM in Oslo in 1936, four years after his death. The original plan provided for two medals to be awarded at every ICM. The number has since grown on occasion to three or four. Nor was there an explicit statement that the prize be awarded only to people who are under 40, although that has always been the case. These medals are retrospective, as are almost all contemporary prizes.

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Rules for the Millennium Prizes

The Clay Mathematics Institute (CMI) of Cambridge, Massachusetts, has named seven “Millennium Prize Problems”. The Scientific Advisory Board of CMI (SAB) selected these problems, focusing on important classic questions that have resisted solution over the years. The Board of Directors of CMI designated a US\$7 million prize fund for the solution to these problems, with US\$1 million allocated to each. The directors of CMI, and no other persons or body, have the authority to authorize payment from this fund or to modify or interpret these stipulations. The Board of Directors of CMI makes all mathematical decisions for CMI, upon the recommendation of its SAB.

The SAB of CMI will consider a proposed solution to a Millennium Prize Problem if it is a complete mathematical solution to one of the problems. (In the case that someone discovers a mathematical counterexample, rather than a proof, the question will be considered separately as described below.) A proposed solution to one of the Millennium Prize Problems may not be submitted directly to CMI for consideration.

Before consideration, a proposed solution must be published in a refereed mathematics publication of worldwide repute (or such other form as the SAB shall determine qualifies), and it must also have general acceptance in the mathematics community two years after. Following this two-year waiting period, the SAB will decide whether a solution merits detailed consideration. In the affirmative case, the SAB will constitute a special advisory committee, which will include (a) at least one SAB member and (b) at least two non-SAB members who are experts in the area of the problem. The SAB will seek advice to determine potential non-SAB members who are internationally recognized mathematical experts in the area of the problem. As part of this procedure, each component of a proposed solution under consideration shall be verified by one or more members of this special advisory committee.

The special advisory committee will report within a reasonable time to the SAB. Based on this report and possible further investigation, the SAB will make a recommendation to the Directors. The SAB may recommend the award of a prize to one person. The SAB may recommend that a particular prize be divided among multiple solvers of a problem or their heirs. The SAB

will pay special attention to the question of whether a prize solution depends crucially on insights published prior to the solution under consideration. The SAB may (but need not) recommend recognition of such prior work in the prize citation, and it may (but need not) recommend the inclusion of the author of prior work in the award.

If the SAB cannot come to a clear decision about the correctness of a solution to a problem, its attribution, or the appropriateness of an award, the SAB may recommend that no prize be awarded for a particular problem. If new information comes to light, the SAB may (but will not necessarily) reconsider a negative decision to recommend a prize for a proposed solution, but only after an additional two-year waiting period following the time that the new information comes to light. The SAB has the sole authority to make recommendations to the directors of the CMI concerning the appropriateness of any award and the validity of any claim to the CMI Millennium Prize.

In the case of the **P** versus **NP** problem and the Navier–Stokes problem, the SAB will consider the award of the Millennium Prize for deciding the question in either direction. In the case of the other problems, if a counterexample is proposed, the SAB will consider the counterexample after publication, and the same two-year waiting period as for a proposed solution will apply. If, in the opinion of the SAB, the counterexample effectively resolves the problem, then the SAB may recommend the award of the Prize. If the counterexample shows that the original problem survives after reformulation or elimination of some special case, then the SAB may recommend that a small prize be awarded to the author. The money for this prize will not be taken from the Millennium Prize Problem fund, but from other CMI funds.

Any person who is not a disqualified person (as that term is defined in section 4946 of the Internal Revenue Code) in connection with the institute may receive the Millennium Prize. Any disqualified person other than a substantial contributor to the institute (as defined in section 507 of the Internal Revenue Code) may also receive the Millennium Prize provided that the directors, upon application for the prize by a disqualified person, shall modify the procedures outlined herein for selecting an awardee so as to assure that the candidate is not present during and does not participate in any deliberations of the Board, the SAB, or any special award committee in connection with making the award and provided further that if an award is made to a disqualified person, the Board shall make public the procedures that are adopted to assure impartiality and to avoid conflict of interest. For purposes of this paragraph, members of the SAB shall be considered “disqualified persons”.

With the one exception in the prior paragraph, all decision-making procedures concerning the CMI Millennium Prize Problems are private. This

includes the deliberations or recommendations of any person or persons CMI has used to obtain advice on this question. No records of these deliberations or related correspondence may be made public without the prior approval of the directors, the SAB, and all other living persons involved, unless fifty years of time have elapsed after the event in question.

Please send inquiries regarding the Millennium Prize Problems to prize.problems@claymath.org.