

# Contents

Preface	xi
<b>Peter S. Ozsváth and Tomasz S. Mrowka</b>	
<b>Introduction</b>	1
1. The subject	3
2. The PCMI Graduate Summer School	3
3. Acknowledgments	5
<b>John Milnor</b>	
<b>Fifty Years Ago: Topology of Manifolds in the 50's and 60's</b>	7
1. 3-dimensional manifolds	9
2. Higher dimensions	10
3. Why are higher dimensions sometimes easier?	14
4. Questions from the audience	15
5. Bibliography	18
<b>Cameron Gordon</b>	
<b>Dehn Surgery and 3-Manifolds</b>	21
Introduction	23
Lecture 1. 3-manifolds and knots	25
1.1. 3-manifolds	25
1.2. Knots	27
1.3. Exercises	29
Lecture 2. Dehn surgery	31
2.1. Overview	31
2.2. Framed surgery on knots on surfaces	34
2.3. Exercises	35
Lecture 3. Exceptional Dehn surgeries	37
3.1. Exceptional surgeries	37
3.2. Lens space surgeries	37
3.3. Seifert fiber space surgeries	38
3.4. Toroidal surgeries	39
3.5. Knots in solid tori	39
3.6. Exercises	41

Lecture 4. Rational tangle filling	43
4.1. Dehn filling	43
4.2. Tangles	43
4.3. Tangles with non-simple double branched covers	47
4.4. Example: the Whitehead link	47
4.5. Exercises	50
Lecture 5. Examples of exceptional Dehn fillings	51
5.1. Some examples	51
5.2. Chain links	54
5.3. Simplicity of the double branched cover	56
5.4. Exercises	57
Lecture 6. Classification of some exceptional fillings	59
6.1. Some classification theorems	59
6.2. Seifert fiber spaces	65
6.3. Methods of proof; non-integral toroidal surgeries	65
Bibliography	69
<b>David Gabai</b>	
<b>Hyperbolic Geometry and 3-Manifold Topology</b>	73
Introduction	75
1. Topological tameness; examples and foundations	76
2. Background material for hyperbolic 3-manifolds	79
3. Shrinkwrapping	82
4. Proof of Canary's theorem	86
5. The tameness criterion	88
6. Proof of the tameness theorem	96
Bibliography	101
<b>John W. Morgan (notes by Max Lipyanskiy)</b>	
<b>Ricci Flow and Thurston's Geometrization Conjecture</b>	105
Introduction	107
Lecture 1. Statement of Thurston's geometrization conjecture	109
1. Prime decomposition	109
2. Examples of geometric manifolds	109
3. Thurston geometrization conjecture	111
Lecture 2. Basics of Ricci flow	113
1. Conjectures on the geometric structure of 3-manifolds	113
2. Review of the Riemann curvature tensor	114
3. The Ricci flow equation	114

4. Examples: Einstein manifolds, Ricci solitons, and gradient shrinking solitons	115
5. Maximum principle	116
6. Consequences of the maximum principle	117
Lecture 3. Blow-up limits and $\kappa$ -non-collapsed solutions	119
1. Definition of blow-up limit	119
2. Criteria for existence of blow-up limits	119
3. $\mathcal{L}$ -geodesics and reduced volume	120
4. Non-collapsing	122
5. Curvature bounds	122
Lecture 4. Structure of $\kappa$ -solutions and canonical neighborhoods	123
1. Gradient shrinking solitons	123
2. Classification of gradient shrinking solitons	124
3. $\kappa$ -non-collapsed solutions	124
4. Canonical neighborhoods	124
5. Existence of canonical neighborhoods for $\kappa$ -solutions	125
6. Canonical neighborhoods for Ricci flows	125
Lecture 5. Structure of regions of large curvature and surgery	127
1. Canonical neighborhoods for regions of high curvature	127
2. Global versions of tubes and caps	127
3. Global structure of the regions of large curvature	129
4. The structure at the singular time	129
5. The surgery process	129
6. Auxiliary Ricci flow to glue in	130
Lecture 6. Ricci flow with surgery and finite-time extinction	131
1. Ricci flow with surgery	131
2. Geometrization	132
Bibliography	137
<b>Marta Asaeda and Mikhail Khovanov</b>	
<b>Notes on Link Homology</b>	139
Introduction	141
Lecture 1. A braid group action on a category of complexes	143
1. Path rings	143
2. Zigzag rings $A_n$	144
3. A functor realization of the Temperley-Lieb algebra	145
4. The homotopy category of complexes	147
5. Braid group representation	148
Lecture 2. More on braid group actions	151
1. Invertibility of $R_i$	151
2. Braid group action on complexes of projective modules $P_i$ and topology of plane curves	151
3. Reduced Burau representation	154

Lecture 3. A Categorification of the Jones polynomial	157
1. The Jones polynomial	157
2. Categorification and a bigraded link homology theory	158
3. Properties and examples	164
Lecture 4. Flat tangles and bimodules	169
1. Two-dimensional TQFTs and Frobenius algebras	169
2. Algebras $H^n$	171
3. Flat tangles and their cobordisms	173
Lecture 5. A homological invariant of tangles and tangle cobordisms	177
1. An invariant of tangles	177
2. Tangle cobordisms	179
3. Equivariant versions and applications	181
Lecture 6. Categorifications of the HOMFLY-PT polynomial	185
1. The HOMFLY-PT polynomial and its generalizations	185
2. Hochschild homology	186
3. A categorification of the HOMFLY-PT polynomial	189
Bibliography	193
<b>Zoltán Szabó</b>	
<b>Lecture Notes on Heegaard Floer Homology</b>	197
Introduction.	199
Acknowledgments.	199
1. Three-manifolds and Heegaard decompositions.	199
1.1. Basic examples.	199
1.2. Heegaard decompositions.	200
1.3. Exercises for Section 1.	202
2. Heegaard diagrams.	202
2.1. Handle decompositions.	202
2.2. Attaching circles.	203
2.3. Examples.	203
2.4. Heegaard moves.	204
2.5. Exercises for Section 2.	205
3. Morse functions.	205
3.1. Exercises for Section 3.	207
4. Symmetric products and generators.	207
4.1. Heegaard generators.	208
4.2. Symmetric products.	208
4.3. Whitney disks.	209
4.4. An obstruction for Whitney disks.	210
4.5. More on homotopy classes and shadows.	210
4.6. Exercises for Section 4.	211

5. Pointed diagrams.	212
5.1. Admissible diagrams.	213
5.2. Two-pointed Heegaard diagrams.	213
5.3. Chain-complexes for pointed Heegaard diagrams.	213
5.4. Chain complexes for knots.	214
5.5. Exercises for Section 5.	214
6. Holomorphic disks.	214
6.1. A mod 2 invariant.	215
6.2. Properties of the Maslov index.	216
6.3. Exercises for Section 6.	217
7. A local formula for the Maslov index.	217
7.1. Exercises for Section 7.	218
8. The chain complex.	218
8.1. Exercises for Section 8.	220
9. Generators and spin-c structures.	221
9.1. Exercises for Section 9.	222
10. Further constructions.	222
10.1. The construction of $HF^+(Y, \mathfrak{s})$ .	222
10.2. Orientations.	223
10.3. Knot Floer homology.	223
10.4. Computations.	225
11. Problems.	225
Bibliography	226
<b>John Etnyre</b>	
<b>Contact Geometry in Low Dimensional Topology</b>	229
1. Introduction	231
2. Contact structures and foliations	233
3. From foliations to contact structures	242
3.1. Part 2 of the proof of Theorem 3.1	244
3.2. Part 1 of the proof of Theorem 3.1	246
4. Taut foliations and symplectic fillings	253
5. Symplectic handle attachment and Legendrian surgery	255
6. Open book decompositions and symplectic caps	258
Bibliography	262

<b>Ronald Fintushel and Ronald J. Stern</b>	
<b>Six Lectures on Four 4-Manifolds</b>	265
Introduction	267
Lecture 1. How to construct 4-manifolds	269
1. Algebraic topology	270
2. Techniques used for the construction of simply connected smooth and symplectic 4-manifolds	270
3. Some examples: Horikawa surfaces	271
4. Complex surfaces	273
5. More symplectic manifolds	273
Lecture 2. A user's guide to Seiberg-Witten theory	275
1. The set-up	275
2. The equations	277
3. Adjunction Inequality	278
4. Kähler manifolds	279
5. Blowup formula	279
6. Gluing formula	279
7. Seiberg-Witten invariants of elliptic surfaces	280
8. Nullhomologous tori	280
9. Seiberg-Witten invariants for log transforms	281
Lecture 3. Knot surgery	283
1. The knot surgery theorem	283
2. Proof of knot surgery theorem: the role of nullhomologous tori	284
Lecture 4. Rational blowdowns	291
1. Configurations of spheres and associated rational balls	291
2. Effect on Seiberg-Witten invariants	293
3. Taut configurations	294
Lecture 5. Manifolds with $b^+ = 1$	297
1. Seiberg-Witten invariants	297
2. Smooth structures on blow-ups of $\mathbf{CP}^2$	299
Lecture 6. Putting it all together: The geography and botany of 4-manifolds	305
1. Existence: the geography problem	305
2. Uniqueness: the botany problem	307
3. Horikawa surfaces: how to go from one deformation type to another	308
4. Manifolds with $c = 9\chi_h$ : a fake projective plane	309
5. Small 4-manifolds	310
6. What were the four 4-manifolds?	311
Bibliography	313