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Introduction

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1. The Summer School and the Theme

The theme of 2010 PCMI Summer School was Mathematics in Image Processing in a broad sense, including mathematical theory, analysis, computation algorithms and applications. Image processing can be viewed as an area in information science where information needs to be processed, extracted and analyzed from visual contents such as imageries or videos. The demands for efficient processing of overwhelming visual information from extensive imaging technologies and the internet are ever growing and challenging. These demands include standard tasks such as compression and denoising as well as high level understanding and analysis, such as recognition and classification. Mathematics has been one of the main driving forces in this field and has provided many powerful tools, such as linear algebra, probability and statistics, harmonic analysis, partial differential equations, optimization, etc. Centered around the theme of mathematics in image processing, the summer school covered a quite wide spectrum of topics in this field through lecture series with associated discussion sessions and computer lab sessions as well as a pretty long list of research seminars.

What makes the summer school more timely and exciting is the very recent advances and developments in mathematical theory and computational methods for sparse representation. Sparsity is an important property that should be investigated and explored for any signal or information. Two key mathematical questions associated with sparse representation are: (1) In what representation are a certain class of signals sparse? For examples, the sparse representation could be in a certain transform domain with a pre-determined basis, such as Fourier or wavelets, or it could be some basis or dictionary computed or learned from a training set; (2) How can one recover the sparse representation effectively from the given measurements? For example, what kind of measurements are desirable and how does one design efficient algorithms to recover the sparse signal? Quite a portion of the lectures and research seminars during the summer school presented state of the art work on sparse representation and compressive sensing from theory to algorithms and applications.

Altogether there were nine lectures series:

- Richard Baraniuk, Rice University: Compressive Sensing: Sparsity-Based Signal Acquisition and Processing
- Antonin Chambolle, École Polytechnique: Total-Variation Based Image Reconstruction

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- Michael Elad, Israel Institute of Technology: Sparse & Redundant Representations: From Theory to Applications in Image Processing
- Anna Gilbert, University of Michigan: A Survey of Sparse Approximation
- Yann LeCun, New York University: Learning Image Feature
- Guillermo Sapiro, University of Minnesota: Dictionary Learning for Efficient Signal Representation
- Zuowei Shen, National University of Singapore: Wavelet and Wavelet Frames in Imaging Science
- Joseph M. Teran, University of California, Los Angeles: Numerical Methods for Elasticity Problems in Biomechanics
- Ross Whitaker, University of Utah: Statistical Models and Methods in Image Analysis

There were also two Clay Senior Scholars in residence during the summer program, Ingrid Daubechies (Princeton University) and Jean-Michel Morel (Ècole Normale Supérieure de Cachanin). Their presence and contributions were invaluable to the whole program. In particular, they gave two very well-received public lectures open to the whole program: Fine Art Meets Mathematics (by Daubechies) and Image Editing with the Poisson Equation: How to teach the Fourier method to undergraduates (by Morel). In addition Tony Derose from Pixar gives a stimulating public lecture about mathematics in computer animation for entertainment industry.

2. Content of the Volume

The content of the current volume consists of three lecture series which give a pretty good glimpse of the topics covered during the summer school. The lecture notes are ordered in the same way as the corresponding lectures were presented during the summer school.

1. *MRA-Based Wavelet Frames and Applications by Bin Dong and Zuowei Shen.* This lecture series includes five lectures which provide a concise, systematic and self contained overview of wavelet frames derived from a multiresolution analysis (MRA-based wavelet frames) and its applications in image analysis and restoration. It starts with an introduction to multiresolution analysis, its definition and approximation theory. Then the concept and concrete construction of tight frames are explained, including the multiresolution based wavelet tight frames and pseduo-spline tight frames. In the later part, applications to image processing based on tight frames are demonstrated. The first part focuses on the mathematical and theoretical foundations of tight frames including fast algorithms for framelet transform. The second part presents some of the very recent developments and applications in image restoration, blind deconvolution and others including a few fast optimization algorithms in these applications. For these applications, sparse representation using tight frames demonstrate various advantages in flexibility and stability due to the redundancy.

2. *Sparse and Redundant Representations Modeling of Images by Michael Elad.* This lecture series consists of five lectures on algorithms, theory and various applications for sparse and redundant representation of images, called *Sparseland.* In particular the author tries to address a few basic questions in the series. For example, how does one (1) find a good dictionary from a set of images that can provide sparse representation for images in a certain class, and (2) find a sparse

representation of an image from a given dictionary. A few formulations and algorithms for learning a dictionary from a training set are presented for question (1). Sparse-coding algorithms and approximation theory are provided for question (2). Various applications and examples are shown to demonstrate the validity of the *Sparseland* assumption and the efficiency of those algorithms introduced in the lectures.

3. *Simulation of Elasticity, Biomechanics, and Virtual Surgery by J. Teran, J. Hellrung, Jr. and J. Hegemann* This lecture series includes three lectures for the simulation of elastic materials characteristic of biological soft tissues. It starts with the modeling of elastic materials and derivations of the corresponding differential equations based on continuum mechanics and elasticity theory. The second lecture is about numerical discretization of those derived differential equations and design of efficient and stable numerical algorithms for computers. The third part provides some more details and further discussions of the implementation.

3. Acknowledgement

We would like to thank all the people who have contributed to the summer program. We are grateful to the lecturers for their effort in providing well prepared and accessible lecture series for this dynamic, broad and exciting field and to the TAs for holding discussion sessions and computer labs. Special thanks goes to those who contributed to this volume. We are sure the PCMI program and this volume will benefit the participants and the community in the years to come.

We would also like to thank Catherine Giesbrecht and Dena Vigil for their invaluable help for organizing the program. The support of the whole PCMI steering committee chaired by Richard Hain is appreciated. Our final thanks goes to John Polking for his help in preparing this volume.