Preface

Number theory is one of the few areas of mathematics for which most problems can be understood by just about anyone, or at least by all those who are familiar with very basic notions of algebra, combinatorics and analysis. Every teacher knows the importance of practicing problem solving: indeed it turns out to be a great way to learn how to reason, no matter the area of mathematics the problems come from. Number theory is quite appropriate for this kind of exercise. For these reasons, a collection of problems in elementary or classical number theory seems in our opinion to be a complementary pedagogical tool for any learning process in mathematics. Moreover, a clever choice of problems can greatly help to raise the curiosity of those who try to solve them.

Unfortunately, very few books are entirely dedicated to problems in number theory. These include the classical work of the great master W. Sierpinski entitled *250 Problems in Elementary Number Theory* and published in Varsovie in 1970, a book which is not well known and unfortunately out of print. Hence, our manuscript does fill an important gap in this area and moreover it has the advantage of having been written to reach a large audience. One can also see it as a practical complement of an earlier book of the authors, that is *Introduction à la théorie des nombres* published by MODULO (2nd edition, 1997), or to any other introductory book in number theory.

Nevertheless, we must admit that our main motivation for writing this book has been our passion for number theory, namely this branch of mathematics which distinguishes itself by its beauty and its numerous mysteries, by its simplicity and its complexity, that is from the proof that there are infinitely many primes to the recently established proof of Fermat's Last Theorem.

This book obviously contains many problems from elementary number theory. Some of these are well known and can be found here and there in introductory books in number theory, while others are not so common. This is namely the case of several problems which we picked from the lesser known manuscript of Sierpinski mentioned above. Our book also contains some problems submitted to the readers of three well known journals: *American Mathematical Monthly*, *Mathematics Magazine* and *The College Mathematics Journal*. Finally, our book contains some 300 new problems never published before.

The choice of problems is obviously subjective; hence, it is no coincidence that the section on arithmetical functions is the longest! In any event, an effort has been made to cover, or at least brush, each of the classical themes of elementary number theory. On the other hand, since more and more students now have to use computers and software to do mathematics, our book can certainly help them in this task. Indeed, many of the problems encourage the reader to use computer software and at times, while searching for a solution, indicate how to write the program that will bring about the solution to the problem.

Although most problems presented here use basic results which can be found in just about any elementary book in number theory, we chose to include a section which provides the basic definitions and the main theorems one needs to handle the various subjects covered in the book. This "tool box" has the advantage that the reader does not have to search here and there for the basic notions needed to solve the problems. Finally, we found it convenient to include in this section a list of the main arithmetic functions with their definitions, as well as a list of the constants and symbols most frequently used in the text.

Our presentation is as follows: the first section provides the basic theory relevant for the understanding and the resolution of the stated problems; the second section gathers the statements of the problems; while the third section lists all the solutions. At the end of the book, the reader will find a bibliography, a terminology index and an index of authors.

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