Contents

Introduction ix

Chapter I. Introduction to Integrable Systems 1
I.1. Symplectic Manifolds 1
I.2. Hamiltonian Systems, Examples 6
I.3. Completely Integrable Systems 10
I.4. Geodesic Flows 13
I.5. Appendix: The Theorem of Darboux 18
Exercises 19

Chapter II. Action–Angle Variables 25
II.1. Hamiltonian Torus Actions 25
II.2. The Arnold–Liouville Theorem 29
II.3. Examples 36
Exercises 40

Chapter III. Integrability and Galois Groups 45
The Arnold–Liouville Theorem, Revisited Through Differential Galois Theory 45
III.1. Variational Equations, Galois Groups, and First Integrals 46
III.2. The Case of a Hamiltonian System 52
III.3. The Case of an Integrable System 55
III.4. Towards Applications: The Reduction 64
III.5. The Example of the Hénon–Heiles System 69
III.6. Appendix: Proof of Ziglin's Lemma 72
Exercises 76

Chapter IV. An Introduction to Lax Equations 81
The Arnold–Liouville Theorem, Revisited Through Lax Equations 81
IV.1. Why Algebraic Curves? 82
IV.2. A Linearization Theorem 85
IV.3. The case of Geodesics on Quadrics 90
IV.4. Appendix: How to Construct Lax Equations (an Overview) 103
Exercises 109

Appendices

Appendix A. What One Needs to Know About Differential Galois Theory 117
A.1. Differential Fields 117
A.2. The Picard–Vessiot Extension 118
A.3. The Galois Group 120
A.4. The Galois Group of a Hamiltonian System is Symplectic 122
A.5. An Example: the Airy Equation 124
Easy Exercises on Differential Galois Theory 128

Appendix B. What One Needs to Know About Algebraic Curves 131
B.1. Complex Curves and Riemann Surfaces 131
B.2. Line Bundles on Curves 133
B.3. Sheaves and Cocycles 135
B.4. The Picard Group, the Riemann–Roch Theorem, the Jacobian 136
Exercises to Check Comprehension of the Definitions 139

Bibliography 141

Index 145