Preface

If you toss a fair coin many times, you would expect the coin to land *heads* as often as *tails*. The goal of this book is to make this intuition precise. As the number of tosses increases, the proportion of heads approaches 1/2, but in what way, how quickly, and what deviations should we expect? *Heads or Tails* is an introduction to probability theory; in particular, it is an introduction to the study of convergence properties of sequences of observations. In this book, I will present an area of mathematics that has both utility and beauty.

Probability theory is the branch of mathematics concerned with the study of random phenomena. A *random phenomenon* is an experiment with an outcome that depends on chance, either because the exact conditions for its outcome are not known or because the randomness of the experiment actually exists. However, we will not discuss the sources of randomness in random phenomena; instead, we will start with a mathematical model of probability. *Heads or Tails* presents an introduction to the mathematical models of these phenomena and to the rigorous deduction of the laws we expect the outcomes of sequences of independent experiments to follow.

While writing this book, I kept the following three points in mind.

1. A freshman- or sophomore-level analysis course is all that is needed to understand the material in this book. In particular, a knowledge of measure theory is not necessary. This book is aimed

toward undergraduate students in math, science, and engineering programs, as well as teachers and all people with a basic knowledge of upper-level mathematics.

2. The level of rigor is that of most mathematics textbooks. The definitions and statements are precise and the proofs are complete.

3. Our discussion will essentially be limited to studying the game of Heads or Tails with a possibly unfair coin: we will study the laws that describe the result of sequences of identical, independent experiments with two possible outcomes. Although this choice may appear too restrictive, the simple game of Heads or Tails actually harbors much of the complexity of the general study of probability. This opinion is evident in Borel's statement that "The game of Heads or Tails, which seems so simple, is characterized by great generality and leads, when studied in detail, to the most sophisticated mathematics."[1]

This book is an invitation to probability theory. Some of the concepts and theorems that it contains are difficult because "elementary" is not a synonym for "easy". The reader should not expect to find strategies for winning the lottery or for maximizing returns from slot machines. On the contrary, the mathematics that we will study shows that the best strategy for such games of chance is abstinence.

Following the excellent suggestion of Pierre Damphousse, the founder and editor of the series in which the French edition of this book appears, I included precise historical background and biographical sketches. A brief bibliography is also included.

To conclude this Preface, I would like to thank the people who helped me pursue mathematical knowledge; the list of colleagues and students who should be thanked is too long to include here. In particular, however, I would like to acknowledge Jean Blanchard and Jean-Pierre Conze, who sparked my interest in mathematics, as well as my friends and colleagues Pierre Damphousse, Marc Peigné, and Elisabeth Rouy, who helped me while writing this short work.

Emmanuel Lesigne

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[1] Émile Borel, *Principes et formules classiques du Calcul des Probabilités*, Chapitre V: *Jeu de pile ou face*; 1924.