
Contents

Preface	ix
Chapter 1. Preliminaries	1
§1. Series	2
1:1. Sums and series.	2
1:2. Some examples	7
1:3. The theorems of Fubini and Tonelli.	8
§2. Probability concepts	10
2:1. Random variables.	10
2:2. Expectations of real-valued random variables that are not necessarily discrete	14
2:3. Second moments, variance, and standard deviation	15
2:4. Independence	16
2:5. Some important distributions	18
§3. Conditioning in the discrete case	19
3:1. Generalities.	19
3:2. Applications in estimating	29
Chapter 2. Markov chains	33

§1. Random walks	33
§2. Discrete time and space Markov chains	42
§3. Further problems on Markov chains	57
Chapter 3. Filtering of discrete Markov chains	61
§1. Filtering	63
§2. Parameter estimation for discrete Markov chains	72
§3. Interpolation for discrete Markov chains	80
3:1. Interpolation equation	80
3:2. Conjugate equations	87
3:3. Other problems related to interpolation	89
§4. Prediction for discrete Markov chains	92
Chapter 4. Conditional expectations	95
§1. L_2 -spaces	95
1:1. Case of finite Ω	95
1:2. General case	104
§2. Definition of conditional expectations	108
2:1. General case	108
2:2. The case of Gaussian random variables	112
§3. Conditional expectations and densities	118
Chapter 5. Filtering of continuous-space Markov chains	127
§1. Filtering of \mathbb{R}^d -valued Markov chains	127
§2. Discrete-time Kalman filter	132
2:1. One-dimensional case	133
2:2. Multidimensional case	137
§3. Linear filtering	147
Chapter 6. Wiener process and continuous time filtering	161

Contents	vii
§1. Introduction	161
§2. Definition and simplest properties of the Wiener process	162
§3. Integration against the Wiener process	168
§4. Recalling power series and systems of ODEs	172
§5. Kalman filter in continuous time	179
Chapter 7. Stationary sequences	191
§1. Definition and simplest properties	191
§2. Spectral densities	197
§3. Filtering stationary sequences	204
§4. The Bochner-Khinchin theorem (optional)	207
§5. The law of large numbers (optional)	210
§6. Spectral representation of stationary sequences (optional)	213
Chapter 8. Prediction of stationary sequences	225
§1. Some properties of rational spectral densities	227
§2. Predicting one step ahead	233
§3. Predicting many steps ahead	240
§4. Dynamic predicting	243
Bibliography	247
Index	249