# Preface

This book is a presentation of the elements of Linear Algebra that every mathematician, and everyone who uses mathematics, should know. It covers the core material, from the basic notion of a finite-dimensional vector space over a general field, to the canonical forms of linear operators and their matrices, obtained by the decomposition of a general linear system into the direct sum of cyclic systems. Along the way it covers such key topics as: systems of linear equations, linear operators and matrices, determinants, duality, inner products and the spectral theory of operators on inner-product spaces. We conclude with a selection of additional topics, indicating some of the directions in which the core material can be applied and developed.

In its mathematical prerequisites the book is elementary, in the sense that no previous knowledge of linear algebra is assumed. It is self-contained, and includes an appendix that provides all the necessary background material: the *very basic* properties of groups, rings, and of the algebra of polynomials over a field. The book is intended, however, for readers with some mathematical maturity and readiness to deal with abstraction and formal reasoning. It is appropriate for an advanced undergraduate course.

As the title implies, the style of the book is somewhat terse. We mean this in two senses.

First, we focus with few digressions on the principal ideas and results of linear algebra *qua* linear algebra. The book contains fewer routine numerical examples than do many other texts, and offers almost no interspersed applications to other fields; these should be adapted to the readership and, if the book is used in a course, provided by the teacher.

Second, the writing itself tends to be concise and to the point, to the extent that some of the proofs might be better described as detailed lists of hints. This is intentional—we believe that students learn more by having to fill in some details themselves.

Besides its style, this book differs from many other texts on the subject in that we try to present the main ideas, whenever possible, in the context of vector spaces over a general field, $\mathbb{F}$, rather than assuming the underlying field to be $\mathbb{R}$ or $\mathbb{C}$. Inner-product spaces, along with the naturally associated classes of self-adjoint, normal, and unitary (or orthogonal) operators, are introduced later than in many books, and the *spectral theorems* for these operators, besides being fundamentally important on their own, also serve here to pave the way for the notions of *reducing* and *semisimplicity* and, eventually, to the general structure theorems—the Jordan form, when the underlying field is algebraically closed, and the corresponding form over general fields.

The text consists of eight chapters and an appendix. These are divided into sections, and further into subsections. Definitions, propositions, examples, etc., are numbered according to the subsection in which they appear, and no subsection has more than one object (definition, theorem, etc.) of each kind. For example, Lemma 1.3.5 is the lemma appearing in subsection 1.3.5, and Theorem 1.3.5 is the theorem appearing in the same subsection. References to the appendix have the form A.x.y (for subsection y of section x, in the appendix).

Exercises appear at the end of sections, and are numbered accordingly, e.g., exercise **ex3.1.2** is the second exercise of section 3.1.

Starred sections, subsections, and exercises contain material that can be skipped on first reading. Several of these sections, as well as parts of the additional topics (in Chapter 8), require some familiarity with basic analysis, e.g., concepts like convergence and continuity.