
Contents

Preface	ix
Chapter 1. Introduction	1
§1.1. The brachistochrone	1
§1.2. The terrestrial brachistochrone	5
§1.3. Geodesics	7
§1.4. Minimal surfaces	14
§1.5. Recommended reading	18
§1.6. Exercises	20
Chapter 2. The First Variation	27
§2.1. The simplest problem	27
§2.2. Euler's approach	28
§2.3. Lagrange's approach	32
§2.4. Recommended reading	45
§2.5. Exercises	46
Chapter 3. Cases and Examples	47
§3.1. Special cases	47
§3.2. Case study: Minimal surface of revolution	52
§3.3. Case study: The brachistochrone	59

§3.4. Geodesics	62
§3.5. Recommended reading	63
§3.6. Exercises	64
 Chapter 4. Basic Generalizations	69
§4.1. Higher-order derivatives	69
§4.2. Case study: The cantilever beam	71
§4.3. Multiple unknown functions	77
§4.4. Lagrangian mechanics	79
§4.5. Case study: The spherical pendulum	85
§4.6. Hamiltonian mechanics	88
§4.7. Ray optics	93
§4.8. Double integrals	96
§4.9. Recommended reading	101
§4.10. Exercises	101
 Chapter 5. Constraints	107
§5.1. Types of constraints	107
§5.2. Lagrange multipliers	114
§5.3. Isoperimetric constraints	117
§5.4. Case study: Queen Dido's problem	120
§5.5. Case study: Euler's elastica	123
§5.6. Holonomic constraints	129
§5.7. Case study: A sliding rod	133
§5.8. Recommended reading	136
§5.9. Exercises	136
 Chapter 6. The Second Variation	139
§6.1. Introduction	139
§6.2. Legendre's condition	142
§6.3. Jacobi's condition	147
§6.4. Case study: The catenoid revisited	160

§6.5. Recommended reading	167
§6.6. Exercises	168
Chapter 7. Review and Preview	171
§7.1. Introduction	171
§7.2. Necessary conditions	172
§7.3. Sufficient conditions	173
§7.4. Two dependent variables	173
§7.5. History and preview	175
§7.6. Recommended reading	176
Chapter 8. The Homogeneous Problem	177
§8.1. Integrals in parametric form	177
§8.2. Euler–Lagrange equations	180
§8.3. The Weierstrass equation	182
§8.4. Case study: The parametric Queen Dido problem	186
§8.5. Recommended reading	189
§8.6. Exercises	190
Chapter 9. Variable-Endpoint Conditions	191
§9.1. Natural boundary conditions	191
§9.2. Transversality conditions	197
§9.3. Focal points	206
§9.4. Case study: Neile’s parabola	206
§9.5. Recommended reading	211
§9.6. Exercises	212
Chapter 10. Broken Extremals	215
§10.1. The Weierstrass–Erdmann corner conditions	215
§10.2. Carathéodory’s indicatrix	224
§10.3. Recommended reading	231
§10.4. Exercises	232

Chapter 11. Strong Variations	235
§11.1. Troubles with weak variations	235
§11.2. Weierstrass's condition	242
§11.3. Case study: Newton's problem	247
§11.4. Recommended reading	254
§11.5. Exercises	254
Chapter 12. Sufficient Conditions	255
§12.1. Introduction	255
§12.2. Fields of extremals	256
§12.3. Hilbert's invariant integral	260
§12.4. Weierstrass's <i>E</i> -function revisited	262
§12.5. The royal road	267
§12.6. Recommended reading	273
§12.7. Exercises	274
Bibliography	277
Index	295