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Inversion Theory and Conformal Mapping

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Contents

Preface

It is rarely taught in an undergraduate, or even graduate, curriculum that the only conformal maps in Euclidean space of dimension greater than 2 are those generated by similarities and inversions (reflections) in spheres. This contrasts with the abundance of conformal maps in the plane, a fact which is taught in most complex analysis courses. The principal aim of this text is to give a treatment of this paucity of conformal maps in higher dimensions. The result was proved in 1850 in dimension 3 by J. Liouville [**22**]. In Chapter 5 of the present text we give a proof in general dimension due to R. Nevanlinna [**26**] and in Chapter 6 give a differential geometric proof in dimension 3 which is often regarded as the classical proof, though it is not Liouville's proof. For completeness, in Chapter 4 we develop enough complex analysis to prove the abundance of conformal maps in the plane.

In addition this book develops inversion theory as a subject along with the auxiliary theme of "circle preserving maps".

The text as presented here is at the advanced undergraduate level and is suitable for a "capstone course", topics course, senior seminar, independent study, etc. The author has successfully used this material for capstone courses at Michigan State University. One particular feature is the inclusion of the paper on circle preserving transformations by C. Carathéodory [**6**]. This paper divides itself up nicely into small sections, and students were asked to present the paper to the class. This turned out to be an enjoyable and profitable experience for the students. When there were more than enough students in the class for this exercise, some of the students presented Section 2.8.

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