Contents

Chapter 1. Introduction 1

Chapter 2. Basic Concepts 13
2.1. Star bodies 13
2.2. Convex bodies 16
2.3. Radon transforms 27
2.4. The Gamma-function 30
2.5. The Fourier transform of distributions 33
2.6. Fractional derivatives 39
2.7. Positive definite distributions 40
2.8. Stable random variables and the function γ_q 44

Chapter 3. Volume and the Fourier Transform 49
3.1. The first examples: hyperplane sections of ℓ_q-balls 49
3.2. A general formula for the volume of hyperplane sections 53
3.3. The parallel section function and the Fourier transform 55
3.4. Parseval's formula on the sphere 62
3.5. Remarks and further results 69

Chapter 4. Intersection Bodies 71
4.1. A Fourier analytic characterization 71
4.2. k-intersection bodies 75
4.3. L_p-balls as k-intersection bodies 80
4.4. The second derivative test 85
4.5. Remarks and further results 91

Chapter 5. The Busemann-Petty Problem 95
5.1. A Fourier analytic solution 95
5.2. How can one make the answer affirmative? 98
5.3. The affirmative part via spherical harmonics 100
5.4. Zvavitch's generalization to arbitrary measures 105
5.5. Remarks and further results 110

Chapter 6. Intersection Bodies and L_p-Spaces 115
6.1. L_p-spaces and positive definite functions 115
6.2. Schoenberg's problems on positive definite functions 121
6.3. Intersection bodies and embeddings in L_p, $p < 0$ 126
6.4. Remarks and further results 139

Chapter 7. Extremal Sections of ℓ_q-Balls 143
7.1. The case of the cube, K. Ball's theorem 143

7.2. The case $0 < q \leq 2$ 147
7.3. Remarks and further results 149

Chapter 8. Projections and the Fourier Transform 151
8.1. A formula for the volume of hyperplane projections 151
8.2. Extremal hyperplane projections of ℓ_q-balls 152
8.3. Projection bodies 155
8.4. The Shephard problem 157
8.5. Remarks and further results 161

Bibliography 163

Index 169