

## Preface

The term “gradient inequality” appears for the first time in the monograph of J. W. Neuberger [91], where the important role of gradient inequalities in proving convergence of solutions to gradient systems is investigated.

The idea that gradient inequalities force convergence in gradient systems was known to S. Łojasiewicz in the 1960s. Łojasiewicz’s celebrated proof [82] of gradient inequalities for analytic gradient maps on finite-dimensional spaces is certainly very complicated, and in fact, the most difficult part in applying the above idea is that of establishing gradient inequalities.

Infinite-dimensional gradient inequalities first appeared in 1983 in the pioneering work of L. Simon [107]. Simon’s gradient inequalities are concerned with analytic gradient maps associated with variational problems; his idea of establishing gradient inequalities has been extended by several authors.

This monograph gives a comprehensive study of gradient inequalities and their applications in proving convergence of solutions to gradient-like systems. Many of the results published here are new, and a lot of them are established by the author himself. The very broad applications fields of these results include the mathematical modeling of physical problems as well as calculus of variations, image processing, geometric evolution problems arising from geometric interests and optimization.

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I dedicate this book to my wife Yan, and my sons Leo and Simon.

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