

CONTENTS

PREFACE TO THE SECOND EDITION	ix
PREFACE TO THE FIRST EDITION	xi
ABOUT THIS BOOK	xiii
INTRODUCTION	1
CHAPTER 1. THE BASIC CLASSICAL NOTIONS	9
1A. Perfect Polish spaces	9
1B. The Borel pointclasses of finite order	13
1C. Computing with relations; closure properties	18
1D. Parametrization and hierarchy theorems	26
1E. The projective sets	29
1F. Countable operations	33
1G. Borel functions and isomorphisms	37
1H. Historical and other remarks	46
CHAPTER 2. κ -SUSLIN AND λ -BOREL	49
2A. The Cantor-Bendixson Theorem	50
2B. κ -Suslin sets	51
2C. Trees and the Perfect Set Theorem	57
2D. Wellfounded trees	62
2E. The Suslin Theorem	65
2F. Inductive analysis of projections of trees	70
2G. The Kunen-Martin Theorem	74
2H. Category and measure	79
2I. Historical remarks	85
CHAPTER 3. BASIC NOTIONS OF THE EFFECTIVE THEORY	87
3A. Recursive functions on the integers	89
3B. Recursive presentations	96
3C. Semirecursive pointsets	101
3D. Recursive and Γ -recursive functions	110
3E. The Kleene pointclasses	118
3F. Universal sets for the Kleene pointclasses	125
3G. Partial functions and the substitution property	130
3H. Codings, uniformity and good parametrizations	135
3I. Effective theory on arbitrary (perfect) Polish spaces	141

3J.	Historical remarks	142
CHAPTER 4.	STRUCTURE THEORY FOR POINTCLASSES	145
4A.	The basic representation theorem for Π_1^1 sets	145
4B.	The prewellordering property	152
4C.	Spector pointclasses	158
4D.	The parametrization theorem for $\Delta \cap \mathcal{X}$	165
4E.	The uniformization theorem for Π_1^1, Σ_2^1	173
4F.	Additional results about Π_1^1	184
4G.	Historical remarks	202
CHAPTER 5.	THE CONSTRUCTIBLE UNIVERSE	207
5A.	Descriptive set theory in L	208
5B.	Independence results obtained by the method of forcing	214
5C.	Historical remarks	215
CHAPTER 6.	THE PLAYFUL UNIVERSE	217
6A.	Infinite games of perfect information	218
6B.	The First Periodicity Theorem	229
6C.	The Second Periodicity Theorem; uniformization	235
6D.	The game quantifier \mathfrak{D}	244
6E.	The Third Periodicity Theorem	254
6F.	The determinacy of Borel sets	272
6G.	Measurable cardinals	280
6H.	Historical remarks	290
CHAPTER 7.	THE RECURSION THEOREM	293
7A.	Recursion in a Σ^* -pointclass	293
7B.	The Suslin-Kleene Theorem	298
7C.	Inductive definability	309
7D.	The completely playful universe	323
7E.	Historical remarks	339
7F.	Results which depend on the Axiom of Choice	341
CHAPTER 8.	METAMATHEMATICS	353
8A.	Structures and languages	355
8B.	Elementary definability	365
8C.	Definability in the universe of sets	371
8D.	Gödel's universe of constructible sets	381
8E.	Absoluteness	390
8F.	The basic facts about L	401
8G.	Regularity results and inner models	416
8H.	On the theory of indiscernibles	446
8I.	Some remarks about strong hypotheses	468
8J.	Historical remarks	473
	THE AXIOMATICS OF POINTCLASSES	475
	REFERENCES	477
	INDEX	491