

Preface

I didn't have time to write you a short letter, so I wrote you a long one instead.

– Samuel Clemens

What Part III is about

I'm taking the time for a number of things

That weren't important yesterday.

– From "Fixing a Hole" by The Beatles

This is Part III (a.k.a. ΔR_{ijkl}), a sequel to Part I ([40]; a.k.a. R_{ijkl}) and Part II ([41]; a.k.a. $\frac{\partial}{\partial t} R_{ijkl}$) of this volume (Volume Two) on techniques and applications of the Ricci flow (we shall refer to Volume One ([42]; a.k.a. g_{ij}) as Volume One).

In Part I we discussed various *geometric* topics in Ricci flow such as Ricci solitons, an introduction to the Kähler–Ricci flow, Hamilton's Cheeger–Gromov-type compactness theorem, Perelman's energy and entropy monotonicity, the foundations of Perelman's reduced distance function, the reduced volume, applications to the analysis of ancient solutions, and a primer on 3-manifold topology.

In Part II we discussed mostly *analytic* topics in Ricci flow including weak and strong maximum principles for scalar heat-type equations and systems on compact and noncompact manifolds, Böhm and Wilking's classification of closed manifolds with 2-positive curvature operator, Shi's local derivative estimates, Hamilton's matrix estimate, and Perelman's estimate for fundamental solutions of the adjoint heat equation.

Here, in Part III, we discuss mostly *geometric-analytic* topics in Ricci flow. In particular, we discuss properties of Perelman's entropy functional, point picking methods, aspects of Perelman's theory of κ -solutions including the κ -gap theorem, compactness theorem, and derivative estimates, Perelman's pseudolocality theorem, and aspects of the heat equation with respect to static and evolving metrics related to Ricci flow. In the appendices we review metric and Riemannian geometry including the space of points at infinity and Sharafutdinov retraction for complete noncompact manifolds with nonnegative sectional curvature. As in previous volumes, we have endeavored, as much as possible, to make the chapters independent of each other.

In Part IV we shall discuss some topics originally slated for Part III such as Hamilton's classification of nonsingular solutions, the linearized Ricci flow, stability of the Ricci flow, the space-time formulation of the Ricci flow, and Type II singularities from the numerical perspective.

Caveat: Many of the chapter numbers of references in Part II to Part III have changed and some of the referred chapters are in Part IV.

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Now that your rose is in bloom, a light hits the gloom on the grey.

– From "Kiss from a Rose" by Seal

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