Preface

In this book, we study existence and asymptotics of rational points on algebraic varieties of Fano and intermediate type. The book consists of three parts. In the first part, we discuss to some extent the concept of a height and formulate Manin's conjecture on the asymptotics of rational points on Fano varieties.

In the second part, we study the various versions of the Brauer group. We explain why a Brauer class may serve as an obstruction to weak approximation or even to the Hasse principle. This includes two sections devoted to explicit computations of the Brauer–Manin obstruction for particular types of cubic surfaces.

The final part describes numerical experiments related to the Manin conjecture that were carried out by the author together with Andreas-Stephan Elsenhans.

Prerequisites. We assume that the reader is familiar with some basic mathematics, including measure theory and the content of a standard course in algebra. In addition, a certain background in some more advanced fields shall be necessary. This essentially concerns three areas.

a) We will make use of standard results from algebraic number theory and class field theory, as well as such concerning the cohomology of groups. The content of articles [**Cas67, Se67, Ta67, A/W, Gru**] in the famous collection edited by J. W. S. Cassels and A. Fröhlich shall be more than sufficient.

Here, the most important results that we shall use are the existence of the global Artin map and, related to this, the computation of the Brauer group of a number field [**Ta67**, 11.2].

b) We will use the language of modern algebraic geometry as described in the textbook of R. Hartshorne [**Ha77**, Chapter II]. Cohomology of coherent sheaves [**Ha77**, Chapter III] will be used occasionally.

c) In Chapter III, we will make substantial use of étale cohomology. This is probably the deepest prerequisite that we expect from the reader. For this reason, we will formulate its main principles, as they appear to be of importance for us, at the beginning of the chapter. It seems to us that, in order to follow the arguments, an understanding of Chapters II and III of J. Milne's textbook [**Mi**] should be sufficient.

At a few points, some other background material may be helpful. This concerns, for example, Artin L-functions. Here, [**Hei**] may serve as a general reference. Precise citations shall, of course, be given wherever the necessity occurs.

A suggestion that might be helpful for the reader. Part C of this book describes experiments concerning the Manin conjecture. Clearly, the particular samples are

chosen in such a way that not all the difficulties, which are theoretically possible, really occur.

It therefore seems that Part C might be easier to read than the others, particularly for those readers who are very familiar with computers and the concept of an algorithm. Thus, such a reader could try to start with Part C to learn about the experiments and to get acquainted with the theory. It is possible then to continue, in a second step, with Parts A and B in order to get used to the theory in its full strength.

References and citations. When we refer to a definition, proposition, theorem, etc., in the same chapter we simply rely on the corresponding numbering within the chapter. Otherwise, we add the number of the chapter.

For the purpose of citation, the articles and books being used are encoded in the manner specified by the bibliography. In addition, we mostly give the number of the relevant section and subsection or the number of the definition, proposition, theorem, etc. Normally, we do not mention page numbers.

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Spring 2014

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