Contents

viii

Part 4: *Postscript*

Foreword to the first edition (1970)

This book is being published in the form in which it was originally planned and written. In some ways, this is not satisfactory: the demands made on the reader are rather heavy, though this is partly also due to a systematic attempt at completeness ('simplified' proofs have appeared of some of my results, but in most cases the simplification comes primarily from a loss of generality).

However, the partly historical presentation adopted here has its advantages: the reader can see (particularly in §5 and §6) how the basic problem of surgery leads to algebra, before meeting the abstract presentation in §9. Indeed, this relation of geometry to algebra is the main theme of the book. I have not in fact emphasised the algebraic aspects of the L-groups, though this is mentioned where necessary in the text: in particular, I have omitted the algebraic details of the calculations of the L-groups, since this is lengthy, and needs a different background. Though some rewriting is desirable (I would prefer to recast several results in the framework suggested in §17G; also, some rather basic results were discovered too late to be fully incorporated at the appropriate points – see the footnotes and Part 4) this would delay publication indefinitely, so it seemed better for the book to appear now, and in this form.

Chapters 0–9 were issued as duplicated notes from Liverpool University in Spring, 1967. They have been changed only by correcting minor errors, adding §1A (which originated as notes from Cambridge University in 1964), and correcting a mistake in the proof of (9.4). Part 2 was issued (in its present form) as duplicated notes from Liverpool University in May 1968. The rest of the material appears here for the first time.

Foreword to the second edition

It is gratifying to learn that there is still sufficient interest in this book for it to be worth producing a new edition. Although there is a case for substantially rewriting some sections, to attempt this would have delayed production indefinitely.

I am thus particularly pleased that Andrew Ranicki has supplemented the original text by notes which give hints to the reader, indicate relevant subsequent developments, and say where the reader can find accounts of such newer results. He is uniquely qualified to do this, and I am very happy with the result.

The first edition appeared before the days of TeX, so the entire manuscript had to be re-keyed. I am grateful to Iain Rendall for doing this efficiently and extremely accurately.

<div align="right">C. T. C. Wall, Liverpool, November 1998.</div>

Editor's foreword to the second edition

The publication of this book in 1970 marked the culmination of a particularly exciting period in the history of the topology of manifolds. The world of high-dimensional manifolds had been opened up to the classification methods of algebraic topology by

- Thom's work on transversality and cobordism (1952)

- the signature theorem of Hirzebruch (1954)

- the discovery of exotic spheres by Milnor (1956).

In the 1960's there had been an explosive growth of interest in the surgery method of understanding the homotopy types of manifolds (initially in the differentiable category), including such results as

- the h-cobordism theorem of Smale (1960)

- the classification of exotic spheres by Kervaire and Milnor (1962)

- Browder's converse to the Hirzebruch signature theorem for the existence of a manifold in a simply connected homotopy type (1962)

- Novikov's classification of manifold structures within a simply connected homotopy type (1962)

- the s-cobordism theorem of Barden, Mazur and Stallings (1964)

- Novikov's proof of the topological invariance of the rational Pontrjagin classes of differentiable manifolds (1965)

- the fibering theorems of Browder and Levine (1966) and Farrell (1967)

- Sullivan's exact sequence for the set of manifold structures within a simply connected homotopy type (1966)

- Casson and Sullivan's disproof of the Hauptvermutung for piecewise linear manifolds (1967)

- Wall's classification of homotopy tori (1969)

- Kirby and Siebenmann's classification theory of topological manifolds (1970).

The book fulfilled five purposes, providing:

1. a coherent framework for relating the homotopy theory of manifolds to the algebraic theory of quadratic forms, unifying many of the previous results;

2. a surgery obstruction theory for manifolds with arbitrary fundamental group, including the exact sequence for the set of manifold structures within a homotopy type, and many computations;

3. the extension of surgery theory from the differentiable and piecewise linear categories to the topological category;

4. a survey of most of the activity in surgery up to 1970;

5. a setting for the subsequent development and applications of the surgery classification of manifolds.

However, despite the book's great influence it is not regarded as an 'easy read'. In this edition I have tried to lighten the heavy demands placed on the reader by suggesting that §§ 0, 7, 8, 9, 12 could be omitted the first time round – it is possible to take in a substantial proportion of the foundations of surgery theory in Parts 1 and 2 and the applications in Part 3 without these chapters.

Readers unfamiliar with surgery theory should have the papers of Milnor [M12], Kervaire and Milnor [K4] at hand, and see how the construction and classification of exotic spheres fits into the general theory. Also, the books of Browder [B24] and Novikov [N9] provide accounts of surgery from the vantage points of two pioneers of the field.

My own experience with reading this book was somewhat unusual. I was a first-year graduate student at Cambridge, working on Novikov's paper [N8], when the book reached the bookshops in early 1971[*]. When I finally acquired a copy, I was shocked to note that the very last reference in the book was to [N8], so that in effect I read the book backwards. The book accompanied me throughout my career as a graduate student (and beyond) – I always had it with me on my visits home, and once my mother asked me: 'Haven't you finished reading it yet?' My own research and books on surgery have been my response to this book, which I have still not finished reading.

Preparing the second edition of the book was an even more daunting experience than reading the first edition. It would be impossible to give a full account of all the major developments in surgery which followed the first edition without at least doubling the length of the book – the collections of papers [C7], [F10] include surveys of many areas of surgery theory. In particular, I have not even tried to do justice to the controlled and bounded theories (Quinn [Q6], Ferry and Pedersen [F9]), which are among the most important developments in surgery

---

[*]I have a vivid memory of telephoning the Foyles bookshop in London in search of a copy, and being directed to the medical department.

since 1970. But it is perhaps worth remarking on the large extent to which the formal structures of these theories are patterned on the methods of this book.

In preparing this edition I have added notes at the beginnings and ends of various chapters, and footnotes; I have also updated and renumbered the references. All my additions are set in italic type. However, I have not modified the text itself except to correct misprints and to occasionally bring the terminology into line with current usage.

A. A. Ranicki, Edinburgh, January 1999.

Introduction

This book represents an attempt to collect and systematise the methods and main applications of the method of surgery, insofar as compact (but not necessarily connected, simply connected or closed) manifolds are involved. I have attempted to give a reasonably thorough account of the theoretical part, but have confined my discussion of applications mostly to those not accessible by surgery on simply connected manifolds (which case is easier, and already adequately covered in the literature).

The plan of the book is as follows. Part 0 contains some necessary material (mostly from homotopy theory) and §1, intended as a general introduction to the technique of surgery. Part 1 consists of the statement and proof of our main result, namely that the possibility of successfully doing surgery depends on an obstruction in a certain abelian group, and that these 'surgery obstruction groups' depend only on the fundamental groups involved and on dimension modulo 4. Part 2 shows how to apply the result. §10 gives a rather detailed survey of the problem of classifying manifolds with a given simple homotopy type. In §11, we consider the analogous problem for submanifolds: it turns out that in codimension $\geqslant 3$ there are no surgery obstructions and in codimensions 1 and 2 the obstructions can be described by the preceding theory. Where alternative methods of studying these obstructions exist, we obtain calculations of surgery obstruction groups; two such are obtained in §12. In part 3, I begin by summarising all methods of calculating surgery obstructions, and then apply some of these results to homeomorphism classification problems: my results on homotopy tori were used by Kirby and Siebenmann in their spectacular work on topological manifolds. In Part 4 are collected mentions of several ideas, half-formed during the writing of the book, but which the author does not have time to develop, and discussions of some of the papers on the subject which have been written by other authors during the last two years.

The order of the chapters is not artificial, but readers who want to reach the main theorem as quickly as possible may find the following suggestions useful. Begin with §1, and read §4 next. Then glance at the statements in §3 and skip to §9 for the main part of the proof. Then read §10 and the first half of §11. Beyond this, it depends what you want: for the work on tori (§15), for example, you first need §12B, (13A.8) and (13B.8).

The technique of surgery was not invented by the author, and this book clearly owes much to previous work by many others, particularly Milnor, Novikov and Browder. I have tried to give references in the body of the book wherever a result or proof is substantially due to someone else.