A FRENCH ANALYTICAL GEOMETRY.

Leçons de Géométrie Analytique. Par MM. BRIOT et BOUQUET.
Revue et annotée par M. APPELL, professeur à la Faculté des
Sciences. Paris, Ch. Delagrave, 1890. 8vo, pp. iii. + 722.

THIS popular French text-book reached its fourteenth
edition in 1890. At that time, as we learn from the preface,
changes in the programmes of the schools and improved
methods of teaching had made a revision of the book advis-
able. This piece of work was done by M. Appell, a mathema-
tician, whose name is as familiar to American students as to
Frenchmen. The bare list of the articles in the book which
he has touched covers a page and a half, and it is safe enough
to say that " *nihil quod tetigit non ornavit.*" A treatise of
this kind is of course more interesting to teachers of element-
ary mathematics than to any one else ; to them even a slight
account of a school book which has achieved great and lasting
popularity in a nation where pure mathematics has flourished
so splendidly and so long, can not fail to prove interesting by
virtue of its subject.

The book opens with a concise notice of the different sys-
tems of plane coordinates, beginning with rectilinear co-
ordinates in general and the particular case of rectangular
axes ; then passing rapidly over polar and bi-polar systems,
and finally giving a notion of coordinates in general. These
notions are all simple enough when presented in the trans-
parent style of the authors ; in fact plane coordinates are so
much simpler than curves drawn on a sphere that it is a
wonder that school books on geography should not give an
account of them before taking up the subject of latitude and
longitude which almost always proves difficult to young
pupils. The writer was once explaining rectangular co-
ordinates at a teachers' institute when one of the members
rose and thanked him for *inventing* them ; he had been try-
ing to teach latitude and longitude without any of the pre-
liminary ideas necessary to an understanding of the matter.
At the close of the first chapter we read, " The representation
of figures by equations is the basis of analytic geometry ; it
allows us to apply the processes of algebra to the study of
figures. In analytic geometry we are concerned with three
fundamental questions : when a figure is defined geometri-
cally, to find its equation ; reciprocally, when the equation is
given, to construct the figure ; finally, to study the relations
which exist between the geometrical properties of the figures
and the analytical properties of the equations."

Chapter II. takes up the first problem ; various loci, in-

cluding nearly all the simple curves whose names are famil-
iar, are defined geometrically and their equations written
down directly, as a mere statement of the definition in the
language of algebra. The curves are drawn and sufficiently
described. In this way the student not only gets a notion
of what a locus is, but, what is far from easy, he comes to
see how an equation, so different in its nature and belonging
to quite another realm of thought, can represent a geometrical
figure, and to look upon equations in x and y as brief state-
ments of the truths of geometry. The mind is put in a con-
dition to understand why the manipulation of an equation
may lead to new facts. Without some such preparation it
can hardly be very profitable to try to prove geometrical
theorems with equations; there can be nothing in the
student's mind corresponding to them, and a gulf which he can
not bridge will exist between the proof and the conclusion.
That the radius of a circle has a constant length is expressed
algebraically by the equation

$$\rho = r \, ;$$

that the sum of the distances of any point on the ellipse
from the foci is constant, by

$$u + v = 2a,$$

and so on ; the polar equations from their simplicity being
first written down. The chapter closes with a list of exercises
of which this is a specimen : " To construct the curve whose
equation in bi-polar coordinates is $uv = a^2$; the distance be-
tween the poles being $2a$." Of course this lemniscate would
be constructed by drawing circles with their centers at the
poles. The student has been told previously how to find
points on the ellipse in the same way. The problems are
mostly too difficult for a beginner.

Chapter III. treats of the fundamental idea of homogeneity.
A function $f\,(a, b, c, \ldots)$, we are told, is homogeneous and
of degree m when $f\,(ka, kb, \ldots) = k^m f\,(a, b, \ldots)$; the
sum, difference, product or quotient of any two homogeneous
functions is homogeneous ; and the same is true of any
power, root or transcendental function of $f\,(a, b, c, \ldots)$;
but the transcendental function must be of degree 0. Thus
$\sin\left(\dfrac{ab}{a^2 + b^2}\right)$ is homogeneous, while $\sin(a + \sqrt{bc})$ is not.
All this is sufficiently clear, and what follows shows its vital
importance at the threshold of analytic geometry.

" When we seek the relations which exist between the
lengths of the various lines A, B, C, ... of a figure, we
imagine these lines referred to a unit of length which is

usually not specified and remains quite arbitrary." Hence
the reasoning which leads to a relation among the lengths of
these lines is independent of any particular unit, and the
relation must subsist whatever be the unit.   In particular it
subsists if the unit be divided by k, that is if the number
expressing each length is multiplied by k ; hence the relation
is homogeneous, or at any rate, if not, then it must break up
into several relations which are each homogeneous.   An
apparent exception occurs when some line of the figure is
taken as the unit of length, but the exception is explained
and the homogeneity reestablished.   " The equations which
the theorems of elementary geometry lead to directly, are
homogeneous. . . . The principle of homogeneity can be
used at every step to verify the algebraic transformations
which have been effected."

The above is a too brief account of a part of this very
elementary and most interesting chapter.   No part of it is
difficult even for a young student, while it opens up to him a
line of thought which he must follow throughout his scientific
studies in whatever direction he turns.   This is the kind of
work which makes mathematicians and scientists, while the
student whose analytic geometry consists only in manipulating
a few equations of which the meaning is but dimly seen, finds
it a barren and useless subject.

Some considerations follow, still of the simplest kind,
which lead to the conclusion that "all rational expressions,
and all irrational expressions containing only square roots, can
be constructed by means of a limited number of right lines
and circles." It is added, but not proved, that no others can.
A little reading of this character would turn the attention of
a goodly number of bright young men who are still at work
upon some of the impossible problems of antiquity, to subjects
more worthy of their abilities.

Book II. opens the study of the right line and circle.   The
treatment of these loci is similar to that in our familiar text
books and is limited to the needs of beginners.   The next
chapter treats of geometric loci in general.   In Book I. the
equations of many loci were obtained simply by writing down
the definition in the language of algebra ; here we obtain the
equations of loci by eliminating one or more variable param-
eters.   The coordinates of a point may be explicitly given as
functions of the parameter a, but more usually they are only
implicitly given in two equations

$$(1) \quad f_1\,(x,\, y,\, a) = 0,$$
$$(2) \quad f_2\,(x,\, y,\, a) = 0.$$

Each value of a gives a pair of curves intersecting in a point

of the locus, and "the equation of the locus is obtained by eliminating the parameter a between the equations (1) and (2)."

Why this should give the equation of the locus is a difficult thing for students to see ; but it is less difficult when properly stated. The result is not a single equation in x and y, but a pair of equations

(3) $\quad f(x, y, a) = 0,$

(4) $\quad F(x, y) = 0.$

which are equivalent to (1) and (2). Any system of values of x, y, a, which satisfies (1) and (2) must satisfy (3) and (4) ; hence equation (4) is satisfied by the coordinates of every point on the locus. Conversely, every system of values of x, y, a which satisfies (3) and (4) must satisfy (1) and (2) ; it follows that (4) is the equation of the locus.

The teacher who reads this book will be everywhere delighted by the careful way in which the raw edges of thought are hemmed down ; no loose threads are left to ravel out and destroy the fabric. It can hardly be doubted that this brave honesty in their elementary school books has much to do with that precision of thought and clearness of expression which makes the works of French mathematicians a perpetual refreshment to the reader. It would be an inquiry worth making, whether students in high schools and normal schools who never intend to enter college do not get more benefit from the study of geometry than any others. With them it is not merely a thing to be crammed for an entrance examination, but a subject to be studied for its educational value. It seems especially disastrous to make analytic geometry, a subject where the preliminary notions are so delicate and beautiful, a thing to be asked questions about when entering college ; inasmuch as the amount of knowledge required can at best be but small, and will almost certainly be acquired under conditions likely to blight future results.

The remainder of the book, while deeply interesting, is more advanced ; and it is not the purpose of this sketch to do more than call attention to those parts whose study may possibly be useful to teachers of classes which are beginning the subject.

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JOHNS HOPKINS UNIVERSITY, *December* 14, 1891.