go reasoning from figures that are purposely drawn false, that
are obtrusively, what all figures are when innerly viewed,
mere diagrams to symbolize the relations under consideration.

I have referred to the importance of connecting geometry
with its practical applications. Here it seems to me that all
our texts, even these recent ones, are weak. The early intro-
duction of trigonometry and mensuration is only one of many
opportunities offered. The triangle of geometry stands for
the triangle of forces, the theory of proportional lines leads
at once to graphical methods of computation, that of inter-
secting lines and planes leads to descriptive geometry and
perspective, while these, in turn, furnish the best possible
introduction to modern projective geometry. Such are
among the many vistas into the surrounding universe that
are to be gained from the highway of geometry. Why should
these be shut out by the close walls of prejudice and custom
on either hand? There is not time for these outlooks! Why,
the way to save time is to make the teaching interesting.
To this end, varied applications and illustrations are the
most effective means.

<div align="right">ELLERY W. DAVIS.</div>

THE MATHEMATICAL CONGRESS AT CHICAGO.

THE mathematical section of the Congress on Mathematics
and Astronomy held in Chicago from August 21st to 26th
was of the highest interest to all present, particularly on ac-
count of the active participation of Prof. Klein, of Göttingen.
Only a brief outline of the proceedings can be presented here,
but it is hoped that a full official report of the proceedings
will ultimately be published.

Monday's session was devoted to preliminary addresses
and to organization, Prof. Klein referring in his introductory
address* to two of the special papers presented:

Gruppentheorie und Krystallographie, by Prof. Schönflies
of Göttingen, and

*Ueber einige mathematische Resultate neuerer astronomischer
Untersuchungen, insbesondere über irreguläre Integrale line-
arer Differentialgleichungen,* by Dr. Burkhardt of Göttingen.

The so-called "structure theory" of crystals deals essen-
tially with a problem of the theory of groups, namely, with
the enumeration of all discontinuous sub-groups which may
be formed from the group of space-movements, combined also
with reflection. Dr. Schönflies has already (1891) made a

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* See this number of the BULLETIN, p. 1.

comprehensive presentation of the theory from this standpoint.

Dr. Burkhardt gives a critical account from a mathematical standpoint of the interesting advances made in the last twenty years by Hill, Gyldén, and others in the field of perturbation calculation. These authors having employed linear differential equations for defining their transcendental functions, their investigations stand in close relations with developments in the theory of these equations made in more recent years from the purely mathematical side.

At the second session (August 22) the following papers were presented :

Ueber die Theorie der algebraischen Invarianten, by Prof. Hilbert of Königsberg, giving an account of the results reached in the investigations on the finiteness of the form systems, in which by the introduction of fundamentally new methods he has passed far beyond the previous standpoint.

Zur Theorie der ganzzahligen algebraischen Gleichungen, by Prof. Weber of Göttingen, giving an elementary proof that there is an infinite number of equations of prime order and with integral coefficients, which in the domain of rationality have no " Affect,"—equations, that is, whose Galois group is the symmetrical group.

Ueber die arithmetisch-algebraischen Tendenzen Leopold Kroneckers, by Prof. Netto of Giessen, illustrating the tendencies followed by Kronecker, particularly in his later years, to reduce the entire field of mathematics to the *relations between integral numbers,* and to consider only such mathematical operations legitimate as involve but a finite number of steps.

Ueber die Reduction der binären quadratischen Formen, by Prof. Hurwitz of Zürich, a derivation, by a particularly simple and elegant geometrical method, of known theorems on the reduction of the binary quadratic forms.

On Fifth-power Numbers, whose Sum is a Fifth Power, by Dr. Artemas Martin of the U. S. Coast and Geodetic Survey, giving special numerical cases.

On the Algebraic Solution of Equations, by Prof. Sawin of Evansville, Wisconsin.

Aeltere und neuere Untersuchungen über Systeme complexer Zahlen, by Prof. Study of Marburg. The author deals with those complex numbers for which the product of two units is a linear combination of the original units ; he gives the historical development of the theory, inviting special attention to the relations of the considerations involved to Lie's theory of continuous transformation-groups.

On the Definitions of the Trigonometric Functions, by Prof. Macfarlane of the University of Texas.

The session of Wednesday opened with a paper on *Modern Graphical Developments*, by President Eddy of the Rose Polytechnic Institute, reviewing the development of graphical methods since the time of Monge, with particular reference to M. Lévy's "La statique graphique et ses applications aux Constructions."

Some Salient Points in the History of Non-Euclidean and Hyper-Spaces were presented by Prof. Halsted of the University of Texas, with an account of the work of an Italian priest, Saccheri, published in 1733, containing a "statement of propositions in Lobatscheffsky's Non-Euclidean Geometry, with their synthetic proof in pure geometric style." [*]

Prof. Study of Marburg then presented a paper on *Some Researches in Spherical Trigonometry*, dealing with the formulæ of the latter from the standpoints of the modern theory of functions and of the theory of groups, and exhibiting close relations to various other branches of mathematics : the theory of orthogonal substitutions, that of desmic surfaces of the fourth order, that of the theta-relations, etc. (See the author's detailed account in the Abhandlungen der sächsischen Akademie, 1893.) This was followed by a paper

On Interpolation Formulæ and their Relation to Infinite Series, by Prof. Echols of the University of Virginia; by a

Résumé de quelques résultats relatifs à la théorie des systèmes récurrentes de fonctions, by Prof. Pincherle of Bologna.

Sur une intégrale définie qui représente la fonction ζ (s) de Riemann, by Prof. Lerch of Prague.

Ueber Eigenschaften von ganzen Zahlen, die durch räumliche Anschauung erschlossen sind, by Dr. Minkowski of Bonn, giving a preliminary account of the present state of the geometrical investigations on parallel grates (Parallelgitter) in space of n dimensions, by means of which he has derived a series of remarkable results in the theory of numbers.

Ueber die nothwendigen und hinreichenden Bedingungen für die Entwickelbarkeit von Functionen einer reellen Variablen nach der Taylorschen Reihe, by Prof. Pringsheim of Munich. The author gives first a review of our present knowledge of criterions of convergence of infinite series, then derives the conditions named in the title.

Consecutive und coincidirende Elemente einer algebraischen Curve, by Prof. Nöther of Erlangen, explaining anew the methods he has employed in the investigation of singular points, and showing the simplicity with which the chief theorems may be expressed.

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[*] See BULLETIN, vol. II., p. 44.—Editor.

Thursday's programme included the following papers :

The Principles of the Elliptic and Hyperbolic Analysis, by Prof. Macfarlane of the University of Texas, in continuation of his earlier paper.

On Weierstrass's Systems of Abelian Integrals of the First and Second Kinds, by Prof. Bolza of the University of Chicago, a precise derivation of certain fundamental theorems, using the methods of Weierstrass.

Fortschritte in der Theorie der linearen Differential-gleichungen, by Dr. Heffter of Giessen, a review of the researches of Fuchs and other German writers during the past five years.

Automorphe Functionen und Zahlentheorie, by Dr. Fricke of Göttingen, giving an account of the relations to arithmetical developments of the numerous discontinuous groups occurring in the theory of automorphic functions, so far as such relations have been investigated by himself and others.

Zur Transformation fünften Grades der hyperelliptischen Functionen erster Ordnung, by Prof. Krause of Dresden.

Sur quelques propositions fondamentales de la théorie des fonctions elliptiques, by Prof. Hermite of Paris, giving a new derivation of the addition theorem of the elliptic functions for any assigned biquadratic form under the radical sign in the elliptic integral.

The following papers were included in the programme for Friday :

On Orthogonal Substitutions, by Prof. Taber of Clark University, giving the determination of all possible real orthogonal substitutions (and of imaginary orthogonal substitutions with two, three, four, or six variables), rationally in terms of the minimum number of parameters; the determination of all symmetric orthogonal substitutions of n variables rationally in terms of $\frac{1}{2}n\,(n-1)$ parameters ; and certain generalizations of Stieltjes' theorem, with an exponential representation of orthogonal substitutions.

On a Quaternary Group of 2520 Linear Substitutions, by Prof. Maschke of the University of Chicago, giving an account of the complete form system of that quaternary group of 2520 substitutions occurring in line geometry.

On Simple Groups, by Prof. Cole of the University of Michigan, describing a new simple group of 504 substitutions of nine letters.

A Doubly-infinite System of Simple Groups, by Prof. Moore of the University of Chicago. The theory of the elliptic modular functions gives a simple group (known since the time of Galois) of $\dfrac{q(q^2-1)}{2}$ substitutions, where q is any prime number greater than 2. By introducing substitutions

with Galois imaginaries, the author obtains an entire series of new simple groups with $\dfrac{q^n\,(q^{2n}-1)}{2}$ substitutions. These results hold also for $q = 2$ if the denominator 2 be omitted. For $q = 2$, $n = 3$, we obtain the group discussed by Prof. Cole.

At the final session (August 26) the following papers were presented :

A Formulary for an Introduction to Elliptic Functions, by Prof. Stringham of the University of California. The author develops a system of formulæ for the reduction of the elliptic integral of the first kind to different normal forms with corresponding functional notations, distinguishing the different types by the particular Cayleyan transformations employed in deriving them. The earlier formulæ of Jacobi and Abel are then derived as well as those of Weierstrass.

A Construction of Galois's Group of 660 Elements, by Mr. Joseph de Perott of Clark University.

Tabellen von endlichen continuirlichen Transformationsgruppen, by Prof. Meyer of Clausthal. One of the most remarkable results of the theory of Lie is that for a given number of variables, for example in the plane, there is only a limited number of distinct types. While Lie, however, has developed these equations only in the abridged infinitesimal form, the present paper contains complete lists of these types in finite form.

Einige Sätze vom Schwerpunkt, and *Der pythagoräische Lehrsatz in mehrdimensionalen Räumen*, by Prof. Schlegel of Hagen.

Considérations générales sur la mesure de la simplicité dans les sciences mathématiques et applications à l'évaluation théorique de la simplicité des tracés géométriques, and *Règle des analogies dans le triangle et transformation continue*, by M. Émile Lemoine of Paris.

Sur l'équation des lignes géodesiques, by Prof. Weyr of Prague.

Nomographie: Sur les équations représentables par trois systèmes rectilignes de points isoplèthes, by M. d'Ocagne of Paris.

Note concerning Arithmetical Operations involving Large Numbers, by Rev. T. M. Pervouchine of Kasan.

Qulques formules relatives aux operations de polaire, by Prof. Capelli of Naples.

Sul moto di rotazione di un corpo rigido attorno ad un punto fisso, by Prof. Paladini of Pisa.

Concerning the Development of the Theory of Groups during the last Twenty Years, by Prof. Klein of Göttingen. From his Erlangen Programme of 1872 (recently translated in

the BULLETIN) as a starting-point, Prof. Klein discusses briefly the development of the theories both of continuous and of discontinuous groups, referring to his recent lecture course, Höhere Geometrie II (soon to be lithographed).

After a unanimous and enthusiastic vote of thanks to Prof. Klein for his efforts in promoting the interest and success of these meetings and for the courtesies extended by him as Prussian commissioner at the Exposition, and after congratulatory remarks by Prof. Story, the section adjourned.

On three afternoons during the sessions of the Congress, Prof. Klein gave highly interesting expositions of the very complete Exhibit of the German Universities at the Liberal Arts Building. Two afternoons were devoted mainly to the explanation and discussion of mathematical models and other appliances, of which an extensive collection had been arranged by Profs. Klein and Dyck. Many of the models were unfamiliar to those present, and the opportunity for their examination was highly appreciated.

The officers of the mathematical section were:

Honorary President, Prof. Klein of Göttingen.

President, Prof. Story of Clark University.

Vice-President, Prof. Moore of the University of Chicago.

Secretary, Prof. Tyler of the Massachusetts Institute of Technology.

Executive Committee, the officers and Prof. White of Northwestern University.

Besides the officers just named the following were among those attending the congress: Profs. Study of Marburg, Halsted and Macfarlane of the University of Texas, Eddy of the Rose Polytechnic Institute, Bolza and Maschke of the University of Chicago, Paladini of Milan, Oliver and Macmahon of Cornell University, Van Vleck of Wesleyan University, Van Velzer of the University of Wisconsin, Beman and Ziwet of the University of Michigan, Smith of the University of Missouri, Fine of Princeton, Waldo of De Pauw University, Merriman of Lehigh University, Loud of Colorado College, Taber and Webster of Clark University, Ely of Vassar, and Messrs. Hulburt of Johns Hopkins University, Holgate of Northwestern University, and Blake of Columbia College.

[For most of the above details in regard to particular papers the writer is indebted to the courtesy of Prof. Klein.]

                                        H. W. TYLER.