

This theorem now admits of manifold applications. It gives us first the transcendency of e , if we regard $C_1, C_2, \dots z_1, z_2, \dots$ as integral rational numbers. It gives further the transcendency of π . For from the equation $1 + e^{i\pi} = 0$ it follows by I. that $i\pi$, and consequently π , can not be algebraic.

It follows further that

For every algebraic number x , except $x = 0$, $X = e^x$ is a transcendental number.

For every algebraic X , except $X = 1$, every natural logarithm $x = \log X$ is a transcendental number.

For every arc which stands in an algebraically expressible relation x to the radius, except $x = 0$, $X = \sin x$ is a transcendental number.

This follows from I, since $2iX = e^{ix} - e^{-ix}$.

The same is true for the other trigonometric lines $\cos x$, $\tan x$, and for the chord $\frac{1}{2} \sin \frac{x}{2}$. To add one more result: The transcendental equation $\tan x = ax$ for a algebraic has, except 0, only transcendental numbers for roots.

SHORTER NOTICES.

A Geometrical Treatment of Curves which are Isogonal Conjugate to a Straight Line with respect to a Triangle. Part I. By I. J. SCHWATT, Ph. D., University of Pennsylvania. Boston, Leach, Shewell and Sanborn, [1895]. 8vo. 6 and 46 pp.

Four points serve to pair off all the points of a plane, if we take for any point its conjugate with respect to all conics through the four points. This is the reversible transformation which Durège (Ebene Curven dritter Ordnung) calls Steiner's Transformation. The absolute or circular points at infinity may be one of the point pairs; the conics are then rectangular hyperbolas, and the four basis points are orthocentric. Any two points OO' forming such a pair, when considered with reference to the diagonal triangle ABC of the four points, are said to be isogonal conjugates, for the reason that AO and AO' make opposite angles with AB and AC .

Transformation by isogonal conjugates is thus a special view of a simple transformation of great importance. In Dr. Schwatt's pamphlet, of which a continuation is in hand, the method is applied in particular to the discussion of the transformation of a straight line, which is, of course

a conic through A, B, C . Since the circular points pass into each other, the line ∞ becomes the circle ABC . Thus the behavior of a line with respect to this circle determines the nature of the conic found by transforming the line. This is, in effect, Dr. Schwatt's starting point. He then discusses in detail the form of the conic for special lines connected with the triangle. As a specimen hyperbola, the isogonal conjugate is taken of that diameter of the circle ABC which passes through the point called by different writers the symmedian point, Grebe's point, and Lemoine's point. It is enough here, to characterize this point K , to say that $ABCK$ is a "Polviereck" or conjugate tetrad of the circle. As a specimen ellipse, the isogonal conjugate of the polar of K as to the circle is considered in detail. This ellipse also has aliases, being called here Steiner's ellipse, but being also called the maximum circum-ellipse.

The properties of Simson's line are also considered, and many details are given as to various points, lines and triangles connected with the given triangle. The method employed is for the most part that of Euclid. The nomenclature used differs a good deal from that of Casey. In one instance the same name is applied differently; Kiepert's hyperbola is with Casey and others a definite hyperbola, but with Dr. Schwatt it is any hyperbola through ABC .

The question of giving references, when dealing with matter both fairly recent and elementary, is an open one. In this case none are given; but it seems a pity not to give a short list of works on the subject, and a critical, or expurgatorial, list would be useful.

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Annuaire pour l'An 1897 publié par le Bureau des Longitudes.
Paris, Gauthier-Villars et Fils.

The *Annuaire* for 1897, which has just appeared, contains 738 pages of tables and descriptive matter and 175 pages of appendices. The astronomical data, which are as usual, very complete, include the comets discovered up to the end of 1895 and the asteroids up to 1896, September 7th; the latter were then 431 in number. A new double star orbit, that of Σ 1879, is introduced, and some new determinations of previously known orbits are added. There are, however, some further changes which might have been made. On p. 159 the value $8''.86$ is used for the solar parallax in preference to the now generally accepted value of $8''.80$ which is merely mentioned in a foot-note. In the same place the mean distance of the earth from the sun is given to *eight*