EULER'S USE OF i TO REPRESENT AN IMAGINARY.

IN the *Introductio in Analysin Infinitorum*, Lausannæ, 1748, Euler uses i to represent an infinitesimal, an infinite, and a positive integer, but in Tom. II, p. 290, we find "Cum enim numerorum negativorum Logarithmi sint imaginarii, * * erit $l. - n$, quantitas imaginaria, quæ sit $= i$."

In a paper *De formulis differentialibus angularibus maxime irrationalibus, quas tamen per logarithmos et arcus circulares integrare licet, M.S. Academiae* [Petropolitanae] *exhibit. die* 5. *maii* 1777, reproduced in Vol. IV., *Institutiones Calculi Integralis*, Petropoli, 1845, pp. 183–194, there occur such passages as "** formulam $\sqrt{-1}$ littera i in posterum designabo, ita ut sit $ii = -1$, ideoque $\frac{1}{i} = -i$." "* * loco cos. φ has duas partes substituamus

$$\tfrac{1}{2}(\cos.\varphi + i\sin.\varphi) + \tfrac{1}{2}(\cos.\varphi - i\sin.\varphi)."$$

"Constat autem esse

$$(\cos.\varphi + i\sin.\varphi)^n = \cos. n\varphi + i\sin. n\varphi."$$

"* * ubi tam x quam y imaginaria involvit, hanc ob rem ponamus brevitatis gratia $x = r + is$, $y = r - is$."

These extracts would seem to dispose of the claim that "Gauss introduced the use of i to represent $\sqrt{-1}$." (See Baltzer, Fink, Wolf, Holzmüller, Thomae, Suter, Harnack, Durège, Chrystal, etc.)

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NOTE ON THE ROOTS OF BESSEL'S FUNCTIONS.

VARIOUS proofs have recently been given of the theorem that between two successive positive (or negative) roots of $J_n(x)$ lies one and only one root of $J_{n+1}(x)$. The following proof (which is contained along with other investigations concerning the roots of Bessel's functions and the hypergeometric series in a paper sent last June to the *American Journal of Mathematics*) is simpler and more elementary than those heretofore given. It depends on the formulæ:

$$\frac{d\,[x^{-n}J_n(x)]}{dx} = -x^{-n}J_{n+1}(x), \quad \frac{d\,[x^{n+1}J_{n+1}(x)]}{dx} = x^{n+1}J_n(x).$$