EULER'S USE OF i TO REPRESENT AN IMAGINARY.

IN the *Introductio in Analysin Infinitorum*, Lausannæ, 1748, Euler uses i to represent an infinitesimal, an infinite, and a positive integer, but in Tom. II, p. 290, we find "Cum enim numerorum negativorum Logarithmi sint imaginarii, * * erit $l. - n$, quantitas imaginaria, quæ sit $= i$."

In a paper *De formulis differentialibus angularibus maxime irrationalibus, quas tamen per logarithmos et arcus circulares integrare licet, M.S. Academiae* [Petropolitanae] *exhibit. die* 5. *maii* 1777, reproduced in Vol. IV., *Institutiones Calculi Integralis*, Petropoli, 1845, pp. 183–194, there occur such passages as "** formulam $\sqrt{-1}$ littera i in posterum designabo, ita ut sit $ii = -1$, ideoque $\frac{1}{i} = -i$." "* * loco cos. φ has duas partes substituamus

$$\tfrac{1}{2}(\cos.\varphi + i\sin.\varphi) + \tfrac{1}{2}(\cos.\varphi - i\sin.\varphi)."$$

"Constat autem esse

$$(\cos.\varphi + i\sin.\varphi)^n = \cos.n\varphi + i\sin.n\varphi."$$

"* * ubi tam x quam y imaginaria involvit, hanc ob rem ponamus brevitatis gratia $x = r + is$, $y = r - is$."

These extracts would seem to dispose of the claim that "Gauss introduced the use of i to represent $\sqrt{-1}$." (See Baltzer, Fink, Wolf, Holzmüller, Thomae, Suter, Harnack, Durège, Chrystal, etc.)

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NOTE ON THE ROOTS OF BESSEL'S FUNCTIONS.

VARIOUS proofs have recently been given of the theorem that between two successive positive (or negative) roots of $J_n(x)$ lies one and only one root of $J_{n+1}(x)$. The following proof (which is contained along with other investigations concerning the roots of Bessel's functions and the hypergeometric series in a paper sent last June to the *American Journal of Mathematics*) is simpler and more elementary than those heretofore given. It depends on the formulæ:

$$\frac{d\,[x^{-n}J_n(x)]}{dx} = -x^{-n}J_{n+1}(x), \quad \frac{d\,[x^{n+1}J_{n+1}(x)]}{dx} = x^{n+1}J_n(x).$$

Applying Rolle's theorem to the first of these formulæ we see that between two consecutive positive (or negative) roots of $J_n(x)$ lies at least one root of $J_{n+1}(x)$; and from the second we see in the same way that between two successive positive (or negative) roots of $J_{n+1}(x)$ lies at least one root of $J_n(x)$. Thus the theorem is proved.

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SHORTER NOTICES.

Introductory Course in Differential Equations for Students in Classical and Engineering Colleges. By D. A. MURRAY, PH.D., Instructor in Mathematics in Cornell University. New York, Longmans, Green, and Co., 1897. Small 8vo, pp. xv+234.

It is the aim of this work as announced in its preface, "to give a brief exposition of some of the devices employed in solving differential equations." As is becoming in a book of elementary character written with this practical end in view, no attempt is made to develop the general theory of differential equations. At the same time, the discussion of the more important cases of "solvable" equations is adequate, and the appended notes contain among other points of theoretical interest a demonstration of the "existence theorem,"—a novel feature in a treatise on differential equations, written in English.

On the whole, the book seems to be an excellent practical introduction to differential equations, containing a well proportioned and suitable treatment of most of the topics which the student needs in his first course in the subject, and of these only, a good variety of exercises, and enough historical and bibliographical notes to suggest further reading.

On the other hand it must be said that the style is not especially attractive and that certain of the discussions are not wholly satisfactory. It will suffice to cite the sections on the symbolic treatment of the linear equation with constant coefficients, which would be clearer were it shown at the outset that the operator D as there involved obeys the fundamental laws of algebra; the chapter on singular solutions, in which it is not noticed that the p-discriminant is in the general case not a singular solution but a locus of cusps; and the demonstration in note H that the necessary condition of integrability of $Pdx + Qdy + Rdz = 0$ is also sufficient.