But the difference of the line-integrals $I_2 - I_1$ is the line-integral around the closed contour 21, so that we have the line-integral of the tangential component of the vector P around the closed contour proved equal to the surface-integral, over a surface bounded by the contour, of the normal component of a vector Ω whose components are

$$\omega_1 = h_2 h_3 \left\{ \frac{\partial}{\partial \rho_2}\left(\frac{P_3}{h_3}\right) - \frac{\partial}{\partial \rho_3}\left(\frac{P_2}{h_2}\right) \right\},$$

$$\omega_2 = h_3 h_1 \left\{ \frac{\partial}{\partial h_3}\left(\frac{P_1}{h_1}\right) - \frac{\partial}{\partial \rho_1}\left(\frac{P_3}{h_3}\right) \right\},$$

$$\omega_3 = h_0 h_2 \left\{ \frac{\partial}{\partial \rho_1}\left(\frac{P_2}{h_2}\right) - \frac{\partial}{\partial \rho_1}\left(\frac{P_1}{h_1}\right) \right\}.$$

The vector Ω is called the *curl* of P.

ON THE STEINER POINTS OF PASCAL'S HEXAGON.

BY DR. VIRGIL SYNDER.

THE proof given by v. Staudt* of the conjugate nature of M, N with regard to the conic for which M, N are associated Steiner points is perhaps rigorous, but unnecessarily long, and the most important statement† is only proved for the particular case in which the two triads of points defining the hexagon are linearly perspective.

He gives a second proof in article 8 of the same paper which is much shorter, but involves imaginary elements.

The following proof is much more simple and direct than either, and shows clearly which of Steiner's points are associated as "Gegenpunkte."

Let A_1, A_2, A_3 and B_1, B_2, B_3 be two triads of points lying on the same conic; these points can be made projective in six ways, namely

$$\begin{pmatrix} A_1A_2A_3 \\ B_1B_2B_3 \end{pmatrix} \quad \begin{pmatrix} A_2A_3A_1 \\ B_1B_2B_3 \end{pmatrix} \quad \begin{pmatrix} A_3A_1A_2 \\ B_1B_2B_3 \end{pmatrix}$$

* Ueber die Steiner'schen Gegenpunkte * * *, *Crelle's Journal*, vol. 62.

† "Weil ferner P, Q harmonïsch getrennt sind durch M und seine Polare * * *."

$$\binom{A_1A_3A_2}{B_1B_2B_3} \quad \binom{A_3A_2A_1}{B_1B_2B_3} \quad \binom{A_2A_1A_3}{B_1B_2B_3}$$

In the first row the three axes of perspective meet in a point M; these axes are also Pascal's lines and can be defined by the same symbols, $\binom{A_1A_2A_3}{B_1B_2B_3}$ now meaning the line joining the points of intersection; A_1B_2 with A_2B_1; A_2B_3 with A_3B_2, etc.

If in the triad $A_1A_2A_3$ each point be joined to the pole of the opposite side, these lines pass through a common point P (Brianchon point), called the pole of $A_1A_2A_3$. Similarly, let Q be the pole $B_1B_2B_3$.

As the position of P and Q does not depend upon the correspondence between the two triads, they are corresponding points in each of the six projections; hence the line joining them is an invariant line and must pass through M, the centre of the involution determined by the first three Pascals. Let F, G be the points in which PQ cuts the conic, and M' the point in which PQ cuts the polar of M; then, from the involutions described,

$$P,\ Q,\ M,\ M',$$
$$F,\ G,\ N,\ N'$$

must be harmonic conjugates.

From the second row of Pascals it follows similarly that

$$P,\ Q,\ N,\ N',$$
$$F,\ G,\ N,\ N'$$

are harmonic conjugates.

This proves that M, N are collinear with P, Q which was first shown by Grossman, in *Crelle's Journal*, vol. 58, who employs an entirely different method.

Further, as two involutions can only have one pair of common elements,

$$M' = N, \quad M = N',$$

hence, Steiner's points of the hexagon A, B are conjugate points with regard to the conic, and divide the segment connecting the poles of the triangle A, B harmonically.

CORNELL UNIVERSITY,
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