SOME RECENT BOOKS ON MECHANICS.*

A Treatise on Physics. By ANDREW GRAY, LL.D., F.R.S. Vol. 1, *Dynamics and Properties of Matter.* London, J. & A. Churchill, 1901. 8vo., xxiii + 688 pp.

Cours de Mécanique. Par CH. CELLÉRIER. Paris, Gauthier-Villars et Fils, 1892. viii + 617 pp.

NEARLY half a century ago Professors William Thomson of Glasgow and P. G. Tait of Edinburgh planned a Treatise on Natural Philosophy. This work, to be published in three volumes, was to contain a thorough discussion of the phenomena of physical science. The plan seems to have been too great to carry through. The first volume was written and published. Later it was reprinted in two large parts—but alone; and the hope of the scientific world that the other two volumes might be forthcoming must needs be given up forever. Now Professor Andrew Gray, Lord Kelvin's successor in the chair of natural philosophy at Glasgow, publishes the first volume of what purposes to be a three volume Treatise on Physics—a sort of junior Treatise on Natural Philosophy as it seems—which, in the words of the author, shall "serve for those who, beginning at the elements of the subject, wish to have in one book an account of theoretical and experimental physics which may be sufficient for most practical purposes of a scientific and technical education."

How like these sentiments are to those which preface the older work! Yet how the conditions have changed! In the sixties there were in English few if any valuable treatises on mechanics or physics; that of Thomson and Tait stood alone. Now there are numerous expositions of every branch of physical science whether experimental or theoretical. Professor Gray himself has provided us with three volumes on Absolute Measurements in Electricity and Magnetism, one volume on Magnetism and Electricity, and a volume on Bessel's Functions and their Applications with the coöperation of Professor Mathews. But the changes in the number of the expositions of science do not compare with the changes in science itself. During the past thirty or forty years so great advances have been made that for one or two authors to compose something which shall be more than a collection of facts and theories, something which

* Continued from the May number of the BULLETIN.

shall carry the reader through a well planned argument from the rudiments to the outmost confines of science and shall place him in immediate touch with research, seems now an impossible task. Consequently Professor Gray limits the scope of his book in the passage we have cited above. As it is, the scope remains sufficiently large, the expanse sufficiently broad and deep, to tax the powers of even the most gifted.

The first volume of this new treatise on physics bears the title Dynamics and Properties of Matter and is intended to set forth all those general principles and theorems of mechanics which may be useful later in the discussions of acoustics, optics, thermodynamics, electricity and magnetism. Some few subjects however, which have a more special than general application, have been held over for later volumes where they will be developed as needed. Such for example are the dynamical theories of osmosis, diffusion, viscosity and the principle of dimensions or dynamical similarity. With these exceptions the volume omits little that could be desired of kinematics, mechanics, hydrodynamics, elasticity, or capillarity.

The mathematical and physical difficulties encountered in reading the book vary so greatly from chapter to chapter, even from section to section, that unless one looks carefully he may be tempted to think that the author has no definite point of view, has merely collected and edited hastily a mass of facts and theorems bearing on physical science. This however is not the case. The author's intention to make his book useful to the greatest possible number has necessarily led him to include a large variety of subjects treated from an elementary standpoint and suited to students or engineers whose knowledge of theoretical mathematics does not extend far beyond a first course in calculus. To have omitted the difficult portions would have meant giving up the aim of the treatise. Consequently we find intermingled sections of all degrees of difficulty. The most difficult are marked by an asterisk which serves not only to warn off the reader but to inform him that what is there developed will not be made use of in any future unmarked sections.

In this manner Professor Gray has produced a work which cannot fail to be of great use. For heretofore if one wished a knowledge of even the elements of dynamics of a particle, rigid dynamics, hydromechanics, and elasticity he would probably be compelled to look into a half dozen books written by different authors, in different notations, and from different standpoints; if furthermore he wished to

learn the more advanced portions of those subjects he would certainly have to lose much time in unnecessarily scattered reading. Fortunately now teachers at the end of a short course in differential and integral calculus can put into the hands of ambitious students this work of Professor Gray which will serve to keep pace for two or three years with the growing mind and enlarging knowledge of the student. To the teacher the book is also valuable owing to its almost encyclopedic nature. It does not do away with special treatises on the various branches of mathematical physics, but it does contain in a connected manner most of the elementary and intermediate parts of these works.

In a short introduction Professor Gray sets forth some statements and points of view that are so true of the work in hand and so untrue of other works as to be worthy of note. After calling attention to the lamentable fact that the study of mechanics has degenerated too much into the mere solving of more or less artificial and tediously algebraic problems, he states that he purposes to deal so far as possible with only such problems as occur and are easily recognizable in nature. He also lays great stress on the point that cause and effect have no place in physics. The force does not cause the motion nor does the motion cause the force—each accompanies the other. The electromotive force and current in a circuit are likewise accompanying, not antecedent and consequent, actions. He shows, however, that the language of cause and effect is often more clear and useful than any other method of speaking and hence ought not to be given up. He finally states that formal definitions and formal logic are not the sole constituents of mathematical physics. More than accurate logic, more than errorless manipulation of analytic processes is essential. The fundamental ideas must be grasped and thoroughly assimilated. These statements the author offers as an apology for what may seem too much profuseness in explanations of the text. The apology is surely unnecessary. The whole book shows the constant and successful attempt to replace formalism by thought. The result is very refreshing inasmuch as the point of view and the little devices of thought are always those of a practical scientist, not those of an ingenious artificer.

After a short chapter of ten pages on the fundamental units, space and time, Chapter II, Kinematics, opens with a discussion of *steps* or vectors and of those properties of centers of gravity which are immediately derivable by means

of steps. Upon page 22 the use of the scalar $i = \sqrt{-1}$ as a quadrantal versor and the multiplication of steps by a general complex scalar are treated. To what purpose this matter has been introduced here is not apparent, unless the author is preparing for the treatment of alternating currents by analytic methods similar to those employed by Professor Steinmetz, of Schenectady. Why no mention of scalar and vector products of vectors is made here or elsewhere in the book is not easy to see. In 1898 Professor Gray states in the introduction to his Treatise on Magnetism and Electricity, "with regard to the mathematical treatment I have, after consideration, decided not to use the vector analysis but to endeavor to insist as far as possible on the physical meaning of the quantities symbolized. Some brevity of expression is no doubt lost by the process but the discussion is on the other hand within the reach of a greater number of readers." Since 1898 Mr. Oliver Heaviside has continued to publish in vector methods, Professor August Föppl has printed his lectures on mechanics in which he constantly employs vectors, and numerous physicists have inclined more and more toward using some form of vector analysis—especially for teaching the elements of mathematical physics. It would seem that in so extended a treatise as this Professor Gray could well make considerable use of vectors, scalar and vector products, and the operators ∇, div., curl, both for abbreviating the analytic expressions and for insisting on the physical meaning of the operations involved. Can it be that Professor Gray has retreated from his position? He has also almost entirely given up that constant insistence on interpretation which was such a successful and valuable feature of his work of 1898. In this respect we think his style has undergone a decided deterioration since writing the unusually clear Treatise on Magnetism and Electricity.

Subtraction of steps and rate of change of steps leads naturally to velocity, acceleration and the hodograph. The hodograph is applied immediately to discuss the motion of a projectile in vacuo and of a point revolving with uniform velocity in a circle. From uniform circular motion there arises a most elaborate discussion of simple harmonic motion —too elaborate for kinematics itself, but amply justifiable by the frequent appearance and great importance of that type of motion in electromagnetic theories. As a generalization, elliptic motion, which is merely the parallel projection of uniform circular motion, is treated. This too is of importance in future theories. The presentation of uniplanar

and spacial kinematics is thorough and admirably brief except for the introduction of the theory of screws. The mention of this subject is quite unnecessary as practically no further use is made of it.

At the beginning of the third chapter, Dynamics, the author takes up the concepts underlying mechanics. He does this not by giving artificial definitions in a deus ex machina fashion, but by pursuing the same line of thought that we suggested in our recent review of Professor Slate's Dynamics. He states Newton's laws of motion in Latin as Newton stated them, translates them, quotes some of Newton's piercing comments which are generally quite overlooked, and makes further comments of his own. The result is a beautiful simplicity and lucidity. No reader can be confused by this part of Professor Gray's work, nor can he fail to be convinced of its essential rightness. In one other point the author sustains us—in reference to the difference or lack of difference between abstract dynamics and geometry. Let us quote a few words from page 107. "Abstract dynamics is a purely ideal science, geometric in a somewhat extended sense caused by the introduction of certain notions not ordinarily employed in purely geometric processes. So long as we confine ourselves to the ideal as we do in geometry there are about it only difficulties of the same kind as we have in geometrical conceptions and these we do not discuss. The special fundamental difficulties arise when the science is applied to the interpretation of nature"

The remainder of the chapter contains the development of the general principles of dynamics—the motion of the mass center, conservation of momentum and of moment of momentum or rate of description of area, the invariable line and plane, momental ellipsoids, pendulums, parallel forces, and impulses. Numerous examples are worked out as illustrations of the principles evolved and methods employed.

The fourth chapter is given over to the discussion and application of Work and Energy. The early introduction of the work done by gravitational forces affords an opportunity for a few preliminary remarks on gravitational potential. The author does not busy himself with computing the work done by numerous system of forces which differ mainly in the degree of complexity of their analytic definition. Instead of this he takes up such real problems as finding the work done in condensing a nebula, in the impact of a jet of particles, in stretching a contractile film,

in blowing a soap-bubble, and in expanding a gas. Thus the student obtains a firm hold on the principle of work and its general applicability to problems he may have to solve later. An elementary discussion of kinetic energy, conservative and dissipative systems, and of friction in particular close the chapter.

The first few sections of chapter V, General Dynamical Methods, deal with the Lagrangian and Hamiltonian equations of motion and with some general dynamical theorems which follow from them. The author once before, in his Treatise on Magnetism and Electricity, has given a treatment of this same subject. In that work thirty pages are devoted to it; in this fewer than half that number. We cannot but regret, and regret deeply, this change. We regarded the earlier presentation of the subject as on the whole the best and clearest to be found in any language. We wished to see the treatment expanded rather than cut down. Space might better be saved in a half dozen other places. The theory of the ignoration of coördinates, of "controllable and uncontrollable" coördinates and of cyclic systems is of the greatest importance in thermodynamics and electromagnetism. Furthermore, discussions of a general nature, such as are set forth in Professor J. J. Thomson's Applications of Dynamics to Physics and Chemistry, are extremely useful in handling systems concerning the minute mechanical construction of which we know nothing and would best assume nothing. This general method of work has not attracted so much attention and consideration as it deserves. References to Professor Thomson's book are all too few and Professor Gray's treatment of that particular phase of dynamical theory in his Magnetism and Electricity was especially welcome. He can have little excuse for omitting the subject here unless he intends to discuss it specially in the later volumes of his work. The complete omission of the study of small oscillations is also difficult to account for. The remainder of chapter V is more elementary and is independent of the foregoing part. The methods of treating dynamics of rigid bodies by Euler's angles and Euler's equations of motion, the motion of gyrostats, and motion relative to the earth are presented.

The next two chapters discuss Statics. In the sixth the analytical conditions of equilibrium of bodies, whether at rest or in steady motion, are derived from the principles of work and energy. In the seventh graphical statics is touched on. The eighth chapter, which treats the equilibrium and motion of a flexible cord or chain, is a welcome

presentation of this interesting and useful branch of dynamics.

Chapters IX and X, each about sixty pages in length, deal with the Statics and Kinetics of Fluids and Hydrostatics. The first of these contains for the most part the theoretical developments and applications of hydrokinetics. The criticisms made upon chapter V apply here with only slightly diminished force. The treatment has been so much condensed and shortened from that in the author's Magnetism and Electricity as to be neither so clear nor so valuable. This shortening is probably due in part at least to the necessity of introducing a mass of more elementary descriptive explanations of different phenomena in the kinetics of fluids. In these explanations the author is quite happy. The chapter on hydrostatics contains few mathematical developments and is confined for the most part to the discussion of such subjects as a student in elementary theoretical physics or in practical laboratory work may need. The treatment includes the theory of buoyancy, hydrometers, barometers, and mercury air-pumps.

Three chapters, XI, XII, XIII, deal with matters immediately connected with the law of universal gravitation. The first, Gravitational Attraction, discusses potential problems, computation of potentials due to certain line, surface, or volume distributions of mass, general theories of potential, the theorems of McLaurin, Charles, and Ivory on the attraction of ellipsoidal bodies, and the method of inversion as applied in potential theories. The author strikes a happy mean between giving too much and giving too little. The second of the three chapters, Astronomical Dynamics, is a welcome presentation of the elements of some theories which it is difficult to find except amid a mass of discouraging detail and small numerical corrections. On the resumption of Gravitational Attraction in the thirteenth chapter the specific attraction due to gravity at the earth's surface is taken up from the standpoint of Cavendish's and other more recent experiments for determining the gravitational constant G.

Chapter XIV on The Tides is as welcome as that on astronomical dynamics; for in it we find a quantity of interesting matter difficult to find elsewhere. After showing that those theories of the tides which are based upon mere changes of the moon's position—the so-called equilibrium theories—are inadequate to explain completely the phenomena observed, the author passes on to the theory of tidal actions considered as due to forced vibrations. As

forced vibrations were not treated in the kinematical or dynamical sections of the book, where they properly belong owing to their fundamental importance not only in tidal theories but in acoustics and electromagnetic vibrations, the author is compelled to make a digression on the subject at this point, after which he proceeds to the discussion of instruments for measuring and predicting tides. At the end of the chapter a few considerations of the changes produced by the tides in the earth-moon system give an opportunity for a passing mention of Professor G. H. Darwin's use of tidal theories in cosmology.

The fifteenth and sixteenth chapters are concerned with Elasticity and Capillarity. The presentation of the theories of elasticity is long, seventy-five pages, and varies greatly in difficulty from advanced mathematical considerations of the general equations of motion of isotropic elastic solids, De St. Venant's theories of torsion, the bending and torsion of thin wires, down through the kinematics of homogeneous strain and the theory of linea elastica to such elementary subjects as vibrations due to torsion in a fiber, flexure of beams, stability of struts, Hooke's law, and impact of elastic balls. The chapter on capillarity is useful owing to the careful arrangement and development of general facts about those simple phenomena of capillary action which are known, vaguely at least, to all students of physics.

The last chapter, the seventeenth, is devoted to the discussion of Measurements and Instruments. It may be interesting to note that Thomson and Tait's Natural Philosophy is concluded in a similar manner. There is nothing remarkable in this chapter which contains merely an exposition of the balance, vernier, spherometer and other simple instruments. In fact we should scarcely have mentioned the chapter at all, had it not given a completeness and finish to the volume and had it not been quite in keeping with the whole. The addition of an extensive index covering ten pages printed in small type greatly enhances the value of the work as a book of reference. But we fail to see the value of adding to a volume which is already becoming bulky a fifteen-page catalogue of the works published by Messrs. J. and A. Churchill—especially inasmuch as that catalogue contains not one work on physics or mathematics.

In conclusion let us state that the author's attempt to write a book from which even the least advanced students may get something, intermediate students a very great deal, and advanced students not a little, seems to have been carried out with more success than we should have thought

possible. We hope the future volumes may be as useful and as well done. We wish success to this junior Treatise on Natural Philosophy. May it attain the completion which was denied to its senior.

Professor Cellérier's Cours de Mécanique is of too old a date to belong properly in this series of reviews. The book, however, came to hand only recently and as it exhibits most clearly the great change which has taken place during the last decade in the point of view adopted in teaching mechanics we may be allowed a few words. Ten years ago when this book appeared mechanics began with statics, continued with dynamics of a particle and rigid dynamics, and concluded with hydromechanics. The amount of space given to statics was great—often greater than that given to any two of the other three. A force was something that tried to produce motion or, if it was fortunate, did produce motion. Everything is now changed. Mechanics begins with dynamics. Statics receives in theoretical works not much more than a passing mention. In fact statics is now unduly slighted where before it was given undue emphasis. Force is the rate of change of momentum—neither the cause of motion nor its result, merely an accompanying phenomenon of motion.

Ten years ago we should have welcomed Professor Cellérier's work and recommended it most heartily for several reasons. First, there is throughout the book a striving against artificiality in problems and points of view. The book is evidently written by one who has a knowledge of physics and engineering in addition to theoretical mechanics. Second, the treatment is well arranged, admirably clear, well adapted to students beginning the subject. Fearing that the student may not be so familiarly acquainted with integral calculus as could be desired, the author evaluates a number of double and triple integrals when considering moments of inertia, and proves Green's theorem. Thirdly, the author introduces the elements of elasticity and hydromechanics into his work. Except in French treatises on mechanics, this was unusual ten years ago. We are happy to say that it is less unusual now, though still not sufficiently customary. The old method of teaching the student a great deal of statics, then a great deal of dynamics of a particle, and leaving rigid dynamics with elasticity and hydromechanics to fit in at the end if there be time must be forsaken. The student gets a better hold on general dynamical principles and a far better preparation

for further work whether in physics or mathematics or theoretical mechanics by beginning with a general course which includes the elements of many branches of mechanics than by beginning with a thorough course in any one branch.

Professor Cellérier's Cours de Mécanique was in many ways an interesting and most valuable book when it appeared. We are inclined even now to recommend it to all who wish to follow the method in general use among engineers—the method of starting mechanics with a discussion of statics.

EDWIN BIDWELL WILSON.

YALE UNIVERSITY,
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KIEPERT'S CALCULUS.

Grundriss der Differential- und Integral-Rechnung. Von Dr. LUDWIG KIEPERT. I. Theil: *Differential-Rechnung.* 9te Auflage. II. Theil. *Integral-Rechnung.* 7te Auflage. Hannover, Helwingsche Verlagsbuchhandlung, 1900–1901.

A WORK which has gone through so many editions in a country where so much attention is given to mathematical instruction should have its merits; nor in this case will the reader fail to discover them. These are clearness of statement, carefully drawn illustrations in great number, minute explanations, warnings against probable mistakes on the part of the student. Nevertheless from the very fact that we have a ninth reproduction of an older work there is a certain clinging to tradition which impairs the usefulness of the book.

For example, it gives the student a better idea both of the theory and the range of application, if the differential and integral calculus be treated simultaneously; moreover, he begins at once to know something of integration, a matter of great practical importance. Again, a subject so rich in applications to the various sciences and industrial arts should be presented along with these applications. Thus can the subject be made alive to the student rather than a dead and tiresome exercise in formula grinding. It may be, however, that a problem book, used in conjunction with the text, will supply this deficiency, and of course every good teacher will add problems of his own.

The first volume has an introduction concerning functions, limits, infinitesimals of various orders, continuity, the binomial theorem for positive integral exponents, geo-