THE APRIL MEETING OF THE CHICAGO SECTION.

A regular meeting of the Chicago Section of the AMERICAN MATHEMATICAL SOCIETY was held on Saturday, April 11, 1903, at the Armour Institute of Technology, in Chicago, opening at 9 A. M. The following members were present:

Professor W. W. Beman, Professor D. F. Campbell, Dr. J. V. Collins, Professor E. W. Davis, Professor B. F. Finkel, Mr. W. B. Ford, Professor Thomas F. Holgate, Professor A. M. Kenyon, Professor Kurt Laves, Dr. A. C. Lunn, Professor H. Maschke, Professor Malcolm McNeill, Professor E. H. Moore, Professor F. R. Moulton, Professor E. B. Skinner, Professor C. A. Waldo, Professor H. S. White, Professor B. F. Yanney, Professor J. W. A. Young, Professor A. Ziwet.

Professor Waldo was elected chairman. The following papers were read:

(1) Mr. OSWALD VEBLEN: "A set of independent axioms of geometry."

(2) Dr. F. R. MOULTON: "The conditions for the convergence of the expressions for the ratios of the triangles when developed as power series in the time intervals."

(3) Professor A. S. CHESSIN: "On the strains in a rapidly rotating disc."

(4) Dr. SAUL EPSTEEN: "On Loewy's fundamental theorem."

(5) Professor PETER FIELD: "On the form of a plane quintic curve with five cusps."

(6) Professor H. S. WHITE: "Polar triangles of a conic and triply tangent conics of a cubic."

(7) Professor H. MASCHKE: "Invariants and covariants of quadratic differential quantics in n variables."

(8) Mr. E. L. HANCOCK: "Asymptotic lines on certain surfaces of revolution."

(9) Professor ARCHIBALD HENDERSON: "On the construction of a double-six."

(10) Professor ARCHIBALD HENDERSON: "On the graphic representation of the straight lines upon the twenty-one different types of the cubic surface."

(11) Mr. O. J. FERGUSON: "Problems in projective geometry."

(12) Professor E. W. DAVIS: "Some groups in the logic of relatives."

(13) Mr. N. J. LENNES: "Theorems on the polygon and the polyhedron."

Mr. Veblen and Mr. Lennes were introduced to the Section by Professor Moore, and Professor Henderson by Professor Maschke; Mr. Ferguson's paper was presented through Professor Davis. In the absence of Professor Chessin and Mr. Hancock their papers were read by title.

The report of the committee on the requirements for the Master's degree, with mathematics as the major subject, was taken up for consideration and after discussion was again laid on the table till the next meeting.

The North Central Association of Science Teachers being in session at Armour Institute, a joint meeting was arranged with the mathematics section of this association, at which Professor W. W. Beman presided and the following papers were read:

"Introduction of physical and other problems into elementary mathematical courses, and the necessary consequences," by Professor C. E. COMSTOCK, of Bradley Polytechnic Institute.

"What is the laboratory method?" by Professor J. W. A. YOUNG, of the University of Chicago.

The discussion of these papers was opened by Miss Long, of the Lincoln, Nebraska, High School.

Abstracts of the papers presented at the Sectional meeting are as follows:

1. The aim of Mr. Veblen's paper, which is intended as a dissertation for the doctor's degree at the University of Chicago, is to present a system of axioms that are at once independent of one another and sufficient for the deduction of the euclidean geometry. The axioms are stated in terms of two fundamental symbols, point and order, and thus follow the trend of development inaugurated by Pasch and continued by Peano, rather than that of Hilbert. All other geometrical concepts, such as line, plane, space, motion, are defined in terms of point and order. In particular, the congruence relations are made the subject of definitions rather than of axioms. This is accomplished by the aid of projective geometry according to the method first given analytically by Cayley and Klein.

2. In rectangular heliocentric coördinates the functions considered in Professor Moulton's paper are

$$(1) \qquad \frac{x_i y_j - y_i x_j}{y_k y_l - y_k x_l},$$

where the x and y are defined by the differential equations

$$(2) \qquad \frac{d^2x}{dt^2} = -\frac{k^2x}{r^3}, \qquad \frac{d^2y}{dt^2} = -\frac{k^2y}{r^3}.$$

All the singularities of x and y considered as functions of t are found and also the conditions under which the denominator of (1) can vanish. Having located all the singularities of the function, the true radius of convergence around any initial point is easily found. The problem is solved in all detail for all classes of conics, and tables are constructed giving numerical results for the various cases which can arise. The relation of the limits in this part of the theory of orbits, to those of the series used in the *Astronomical Journal*, No. 510, is given, and an exhibition of the corresponding question in Laplace's method. The paper will be published in the *Astronomical Journal.*

3. Professor Chessin finds expressions for the displacements of a particle in a thin rotating disc in terms of the coördinates of the particle and certain constants determined from the surface conditions.

5. Del Pezzo has shown how it is possible to obtain the equation of a five cusped quintic curve by inverting a quartic which has two cusps and which is inscribed in and circumscribed about the fundamental triangle. The equation of the quintic is written in the form $C_4^2 - \phi^3\phi_1 = 0$, where C_4 is a quartic and the ϕ's are conics; the constants being so chosen that x, y, z are factors of the equation. Professor Field divides the plane into regions by drawing the curves C_4, ϕ, ϕ_1, and by their aid obtains the form of the curves represented by the above equation.

6. A plane cubic may be so situated with reference to a conic as to admit an inscribed triangle, self-conjugate with respect to the conic, and having one degree of freedom. It is shown by

Professor White that the vertices of such a triangle will be contacts of a tritangential conic. There are 3 systems of such conics, corresponding to the 3 pro-Hessians of the cubic. The tritangential conics of a simply infinite system like that mentioned above are poloconics of a simple infinity of straight lines, with respect to some one of the 3 pro-Hessians. It is found that these lines form a pencil, all passing through one common point. There is thus shown the existence, under special conditions, of a covariant point in the system of a conic and a cubic.

7. Professor Maschke's paper will be published in the *Transactions*.

8. Mr. Hancock considers two surfaces of revolutions S and S_1, generated by the curves C and C_1. C_1 is formed by taking on the tangents to C a length equal to l^{-1} times the length of the tangent. The generating curves then are

$$z = f(u), \quad z_1 = (l-1)\, u_1 f'\ (lu_1) + f\ (lu_1),$$

and the equation of transformation from S to S_1, $u = lu_1$, $v = v_1$.

The condition that the asymptotic lines on S go over into the asymptotic lines on S_1 by the transformation is $D_1/D_1'' = e^3\, D/D''$. This is seen in particular when the generating curves are $z = u^{\pm n} + C_2$ and $z_1 = k_1 u_1^{\pm n} + C_2$. The paper will be offered to the *American Journal of Mathematics* for publication.

9. A double-six, in the Schläfli-Cayley notation, is represented by the scheme

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1' & 2' & 3' & 4' & 5' & 6' \end{pmatrix}$$

and is defined to be two systems of six lines each, such that no two lines of the same system intersect, but that each line of the one system intersects all but the corresponding line of the other system. In Professor Henderson's paper there is given a simple method of constructing a double-six, viz., if we select the equations of the five lines $1'$, $3'$, $4'$, $5'$, $6'$, such that they do not intersect, but such that they are all met by the same straight line 2, whose equations are also known, then the equations of the remaining six lines 1, 3, 4, 5, 6 and $2'$, are uniquely determined. Using quadriplanar coördinates and properly choosing the constants, a thread model of the configuration was very easily constructed.

10. Cayley in his Memoir on cubic surfaces (Collected mathematical papers, volume VI (1868), pages 359–455), has given a treatment of the twenty-three types of the cubic surface, the division depending upon the nature of the singularities, and not upon the reality of the lines. The equations there given are canonical forms in quadriplanar coördinates. Two of the surfaces however are scrolls, in which there is no question of the twenty-seven lines.

In Professor Henderson's second paper there is developed a simple method of graphically representing the lines on the remaining twenty-one types of the cubic surface, given by the above-mentioned canonical forms. By this method the mutual relations of the lines, as well as their relation to the fundamental tetrahedron, are shown and the results obtained are equally adapted to the construction of thread models or perspective drawings of the various configurations.

The paper was illustrated by perspective drawings in colors of the lines upon the twenty-one types of the cubic surface, and incidentally there was shown a graphic representation of the projection of the lines upon a cubic surface with a conical point into the Pascal configuration.

11. Mr. Ferguson's paper contained demonstrations of the following theorems : 1° If a range of points μ is projective to a pencil of rays S, and if through the points of μ there are drawn rays perpendicular to the corresponding rays of S, these generate a ruled surface of the fourth order. 2° In two projective sheaves of rays the common perpendiculars of corresponding rays generate a ruled surface of the sixth order.

12. Professor Davis established a one to one correspondence between the different ways in which $3, 4, \cdots, n$ relatives can be connected by logical multiplication and addition, and the substitutions on $3, 4, \cdots, n$ letters.

13. Under formal definitions of polygon and polyhedron, Mr. Lennes proved certain theorems on the partition of the plane and space.

THOMAS F. HOLGATE,
Secretary of the Section.