ably clear and thorough account of the special cases that occur in the solution of linear systems and their dependence, second, the treatment of Sylvester's dialytic method of elimination in which not only the necessity but the sufficiency of the condition that two equations have λ common roots is derived — a point in which most elementary texts are painfully deficient. A brief and hence necessarily unsatisfactory note appears in the appendix on the development of the properties of determinants by means of alternate numbers, a method which is extremely rapid, and with somewhat mature students quite teachable.

No geometrical applications are given, except in isolated examples. Problems are abundant, well graded and not trivial. Not only is the book sufficiently elementary to warrant its wide use as a text-book, but its many pedagogical excellencies make it very valuable to a teacher of the subject.

H. E. HAWKES.

Liniengeometrie mit Anwendungen. Band I. By Dr. KONRAD ZINDLER, Professor an der Universität Innsbruck. Sammlung Schubert, XXXIV. Leipzig, G. J. Göschen, 1902. 8vo., viii + 380 pp., 87 figures.

IN this systematic exposition of line geometry, both synthetic and analytic methods are employed. It is the first book of this character since the appearance of Plücker's original treatise.* In fact, Sturm's Liniengeometrie in three volumes (1892, 1893, 1896) is purely synthetic; while other books, such as Koenigs's Géométrie réglée, treat only special parts of the subject.

Line geometry treats of manifoldnesses of straight lines in space. The names ruled surface, congruence, complex are applied to a one-fold, two-fold, three-fold system, respectively.

The present (first) volume treats chiefly of linear complexes and congruences and their applications; the second volume will be concerned chiefly with algebraic line configurations of higher than the first degree. The book is intended as a text-book; eighty-two exercises are scattered through the book, hints for the solution of certain of which are given in the appendix, with references for the others.

Chapter I treats of the Nullsystem and the Strahlengewinde. Let all the points of space be subjected to a given screw motion

* Neue Geometrie des Raumes, gegründet auf die Betrachtung der geraden Linie als Raumelement, I, 1868 ; II, 1869.

and let each point be made to correspond to that plane through it which is normal to the path of the motion. Inversely, to every plane corresponds a point. This one-to-one geometric correspondence is called a Nullsystem. If a point describes a straight line, the corresponding plane turns about another line, and inversely. Moreover, the correspondence preserves incidences. Hence a Nullsystem is a correlation in the sense of projective geometry. The totality of normals, at all points of space, to the path of a screw motion forms a Strahlengewinde; it is composed of ∞^3 straight lines; every point of space and every plane of space is the bearer of a flat-pencil of these lines. A simple visualization of the lines of a Strahlengewinde may be obtained by taking certain ∞^2 tangents to each of ∞^1 coaxial right circular cylinders.

Chapter II treats at length (pages 31–71) of applications to the theory of motion, mechanics and graphic statics. Chapter III treats in fifty-five pages the subject of line coördinates and the coördinates of a segment (Stab) defined by the length and the position of the line on which the sect lies. By way of introduction, the chief properties of homogeneous point and plane coördinates are given without proof.

Chapter IV treats of linear configurations of lines with applications to mechanics. A linear homogeneous equation between the six segment coördinates defines a linear complex. It is shown to be identical either with a Strahlengewinde or else with the totality of lines intersecting a given one. Hence the linear complex can be represented by a linear homogeneous equation in line coördinates. Its coefficients define a correlation in space for which every point lies in its corresponding plane; inversely, every such correlation may be defined by a Strahlengewinde. The section of two linear complexes is a congruence of the first order and first class, called (after Sturm) a net of lines. The properties of nets, including a complete classification, are given on pages 160–197.

Chapter V (pages 203–278) treats of imaginary elements, and their geometric interpretation, including a historical and logical discussion. The final chapter, VI, relates to manifoldnesses of linear complexes and applications to mechanics and the theory of motion. Among the topics treated may be mentioned the cylindroid (a ruled surface of the third order), general complex coördinates, and Klein line coördinates.

L. E. Dickson.