THE OCTOBER MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

The one hundred and twentieth regular meeting of the AMERICAN MATHEMATICAL SOCIETY was held in New York City on Saturday, October 29, 1904, extending through the usual morning and afternoon sessions. The following twenty-four members were present:

Professor E. W. Brown, Professor F. N. Cole, Dr. W. S. Dennett, Dr. William Findlay, Professor T. S. Fiske, Dr. A. S. Gale, Mr. S. A. Joffe, Dr. Edward Kasner, Dr. G. H. Ling, Mr. L. L. Locke, Dr. Max Mason, Professor W. H. Metzler, Professor James Pierpont, Miss Amy Rayson, Dr. I. E. Rabinovitch, Professor Charlotte A. Scott, Professor D. E. Smith, Mr. E. R. Smith, Professor Henry Taber, Professor H. D. Thompson, Professor E. B. Van Vleck, Professor J. M. Van Vleck, Mr. H. E. Webb, Miss E. C. Williams.

The President of the Society, Professor Thomas S. Fiske, occupied the chair. The Council announced the election of the following persons to membership in the Society: Mr. R. P. Baker, Union Academy, Anna, Ill.; Dr. W. H. Bussey, Evanston, Ill.; Mr. A. M. Curtis, State Normal School, Oneonta, N. Y.; Professor G. R. Dean, University of Missouri; Professor R. R. Fleet, William Jewell College, Liberty, Mo.; Professor E. D. Grant, Michigan College of Mines; Mr. J. E. Higdon, Shortridge High School, Indianapolis, Ind.; Dr. L. C. Karpinski, University of Michigan; Dr. O. C. Lester, Yale University; Mr. Arthur Ranum, University of Wisconsin; Mr. C. H. Sisam, U. S. Naval Academy; Miss Adelaide Smith, Huguenot College, Wellington, Cape Colony; Professor C. M. Snelling, University of Georgia; Professor Eduard Study, University of Bonn. Five applications for membership in the Society were received.

A list of nominations for officers and other members of the Council was adopted and ordered placed on the official ballot for the annual election. A committee consisting of Professor Legras and Dr. Kasner was appointed to audit the Treasurer's accounts for the year 1904.

The following papers were presented at this meeting:

(1) Dr. EDWARD KASNER: "Contact transformations and related systems of curves."

(2) Dr. MAX MASON: "The doubly periodic solutions of $\Delta u + \lambda A(x, y)u = f(x, y)$."

(3) Professor E. B. VAN VLECK: "A proof of some theorems on pointwise discontinuous functions."

(4) Professor HENRY TABER: "Hypercomplex number systems."

(5) Mr. J. C. MOREHEAD: "Note on a theorem of Lucas on Fermat's numbers."

(6) Professor E. W. BROWN: "On the completion of the new lunar theory."

(7) Professor VIRGIL SNYDER: "Quintic scrolls having a tacnodal or an oscnodal conic."

(8) Professor G. A. MILLER: "Groups of subtraction and division."

Mr. Morehead's paper was communicated to the Society through Professor Pierpont. In the absence of the authors, Mr. Morehead's paper was read by Professor Pierpont, and the papers of Professor Snyder and Professor Miller were read by title. Abstracts of the papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. A contact transformation of the plane converts the ∞^2 points into ∞^2 curves C_1, and at the same time converts ∞^2 curves C into points. Two doubly infinite systems of curves C and C_1 which can be obtained in this way are termed *related* in Dr. Kasner's paper. They may be represented by a single equation $\Omega(x, y; x_1, y_1) = 0$, involving two sets of point coördinates, the system C arising when x_1, y_1 are regarded as parameters and the system C_1 when x, y are the parameters. All the systems related to a given system are equivalent under the group of all point transformations. When a system admits a group of point transformations the same is true of any related system. The general discussion is supplemented by a detailed discussion of related systems of circles, especially of systems related to themselves.

2. Picard has studied the doubly periodic solutions of

$$\Delta u - u = 0 \tag{1}$$

which possess logarithmic singularities. No periodic solution, except zero, of this equation exists which is everywhere finite. In Dr. Mason's paper the existence of everywhere finite doubly periodic solutions of

$$(2) \qquad \Delta u + \lambda A(x, y)u = f(x, y)$$

is considered, where λ is a parameter, A and f are doubly periodic. A doubly periodic solution of (1), given by Picard, is used as a Green's function to reduce the differential equation (2) to a functional equation of a type studied by Fredholm. Theorems are deducted from Fredholm's results, and from the consideration of equation (2), with right member zero, as the necessary condition of an isoperimetric calculus of variations problem. There exists an infinite series of special values of λ for which the homogeneous equation possesses finite doubly periodic solutions. These solutions are the solutions of the minimum problems. The equation (2) possesses a unique doubly periodic solution for any value of λ not a special value, and has a doubly periodic solution for a special value of λ when and only when $f(x, y)$ satisfies n integral conditions, where n is the number of linearly independent solutions of the homogeneous equation for this value of λ. The paper will appear in *Liouville's Journal.*

3. The paper of Professor Van Vleck contained a proof of some six or seven theorems on pointwise discontinuous functions, due mostly to Baire (*Annali di Matematica,* 1899). The demonstration which had been previously given by Baire was based upon the concept of "upper continuity." The introduction of this concept is, however, unnecessary, and the proof was reduced by Professor Van Vleck to yet simpler and more fundamental principles. It was also shown that in one of the theorems more restriction had been imposed upon the function involved than was necessary to establish the conclusion.

4. Professor Taber's paper showed how to reduce to the determination of certain nilpotent number systems the problem of the enumeration of all types of number systems in n variables (for an arbitrary value of n), into one or other of which any given number systems in n variables can be transformed by a transformation rational in the domain of the constants of

multiplication of the given system. It contained also generalizations with respect to a restricted domain of rationality of Cartan's theorems given in the *Comptes Rendus* for 1897. This paper appeared in part in the *Transactions* for October.

5. Mr. Morehead pointed out a singular error in the theorem announced by Lucas on page XII of the preface of his Théorie des nombres: In order that the number $2^{2^n} + 1$ be prime, it is necessary and sufficient that it should be a divisor of the number $3^{2^{2^n}} + 1$. This statement which is quoted without correction in Pascal's Repertorio, volume 1, page 551, and again in the German edition of the latter work, volume 1, page 516, is obviously at odds with the Fermat law $3^{p-1} \equiv +1 \pmod{p}$.

6. Professor Brown's paper consisted of a general account of the work required to solve that part of the lunar problem which depends on the sun's action. The solution has been under way for the last twelve years and it has just been completed.

7. In volume 9 of the BULLETIN, Professor Snyder discussed five types of quintic scrolls having three double conics. The present paper shows that two or all three of the double conics may become consecutive, and derives the equation of each form. When two conics approach coincidence, the two generators through each point of one of them must lie in a plane containing the tangent to the conic at that point. The point correspondence on the conic is discussed and compared with the Noether-Wiman depiction of the surface on a twisted cubic curve. Two types exist with a tacnodal conic, and one with an oscnodal conic. The paper will appear in full in the BULLETIN.

8. Let x_1, x_2 represent any two fixed numbers, while n represents an arbitrary number. The two operations $x_1 - n$, x_2/n generate a group of finite order only when x_1 and x_2 satisfy certain conditions. In the four cases when $x_1 = 0$, $x_1^2 = x_2$, $x_1^2 = 2x_2$, or $x_1^2 = 3x_2$ the corresponding groups are respectively of orders 4, 6, 8, 12. All of these groups are of the dihedral rotation type and the two operations $x_1 - n$, x_2/n cannot generate any other group which transforms every rational point of the plane into a rational point. Hence these four groups are of fundamental importance in the theory of point transforma-

tions of the plane. Every dihedral rotation group can be represented as a subtraction and division group, and the direct interpretation of these operations is always a dihedral rotation group. The operations may be resolved, when complex numbers are employed, so as to lead to other interesting groups.

F. N. COLE,
Secretary.

THE FUNDAMENTAL CONCEPTIONS AND METHODS OF MATHEMATICS.

ADDRESS DELIVERED BEFORE THE DEPARTMENT OF MATHEMATICS OF THE INTERNATIONAL CONGRESS OF ARTS AND SCIENCE, ST. LOUIS, SEPTEMBER 20, 1904.

BY PROFESSOR MAXIME BÔCHER.

I. *Old and New Definitions of Mathematics.*

I am going to ask you to spend a few minutes with me in considering the question: what is mathematics? In doing this I do not propose to lay down dogmatically a precise definition; but rather, after having pointed out the inadequacy of traditional views, to determine what characteristics are common to the most varied parts of mathematics but are not shared by other sciences, and to show how this opens the way to two or three definitions of mathematics, any one of which is fairly satisfactory. Although this is, after all, merely a discussion of the meaning to be attached to a name, I do not think that it is unfruitful, since its aim is to bring unity into the fundamental conceptions of the science with which we are concerned. If any of you, however, should regard such a discussion of the meaning of words as devoid of any deeper significance, I will ask you to regard this question as merely a bond by means of which I have found it convenient to unite what I have to say on the fundamental conceptions and methods of what, with or without definition, we all of us agree to call mathematics.

The old idea that mathematics is the science of quantity, or that it is the science of space and number, or indeed that it can be characterized by any enumeration of several more or less heterogeneous objects of study, has pretty well passed away among those mathematicians who have given any thought to