the working out of what seem at first only interesting particular cases that enables us to make real progress in the general theory. In spite of the deductive form which we give our results, we can as little dispense with the inductive method in mathematics as in any other branch of human learning.

The week ended with further attractive excursions for those whose departure was not too urgent.

The writer acknowledges with cordial appreciation the courteous response of participants in the Congress to his request for abstracts of their papers. By the kind coöperation of Dr. E. B. Wilson, of Yale University, the abstracts of the papers read before the several sections will appear later in the BULLETIN.

H. W. TYLER.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
*November*, 1904.

---

## THE HEIDELBERG CONGRESS: SECTIONAL MEETINGS.

The several sections of the Congress at Heidelberg met on Tuesday afternoon, August ninth, to organize and arrange programs. For each section there had been previously appointed two or three presidents, of whom one delivered an address, usually merely a few words of welcome and of pleasurable anticipation of the scientific value of the work of the section. In the cases of von Brill and Klein, however, the address widened out into as truly a scientific communication as any presented at the subsequent meetings. Instead of expecting the presidents to take the chair at all the meetings of their sections, the excellent custom of selecting honorary chairmen from among those present was adopted.

The meetings were held Wednesday and Friday mornings. Professor Tyler, representing the AMERICAN MATHEMATICAL SOCIETY, inserted in the *Tageblatt* a statement that the Society would be glad to receive for publication in the BULLETIN abstracts of the various papers, and sent to each speaker whose address could be obtained a circular letter repeating the request. The response was general and cordial, and we have here especially to thank the members of the congress for their attitude. The length of the abstracts was to be 100 words, but many considerably exceed this limit. The editor, however, has refrained from cutting down the report of the author except in such few instances as have seemed necessary, and possible without injuring the value of the abstract. In one or two cases also editorial exigency has dictated some omission for the sake of avoiding possible error. It is to be regretted that abstracts of some of the more important papers, such as those of Hilbert, on the foundations of arithmetic, and König, on the continuum, have not been received. We may note, however, that the general secretary of the congress, Professor Krazer, is pushing the publication of the proceedings as rapidly as possible, and we may thus hope to have in full all these valuable papers, some of which by the kindness of their authors are here given in brief abstract.

### Section I. *Arithmetic and Algebra.*

Presidents: Lüroth and Kneser. Chairmen: Selivanov and Netto.

(1) GORDAN (Erlangen): "Ueber Gleichungen sechsten Grades."†

(2) KÖNIG (Budapest): "Beweis, dass das Continuum keiner wohlgeordneten Menge äquivalent sein kann."

(3) CAPELLI (Naples): "Ein Beitrag zur niederen Zahlentheorie (inbesondere zum Fermat'schen Satze)." †

(4) HOČEVAR (Graz): "Ueber die Bestimmung der linearen Teiler einer algebraischen Form." †

(5) GULDBERG (Christiania): "Ueber lineare Differenzengleichungen." †

(6) MINKOWSKI (Göttingen): "Zur Geometrie der Zahlen."

(7) HILBERT (Göttingen): "Ueber die Grundlagen der Arithmetik."

(8) VORONOÏ (Warsaw): "Sur une propriété du discriminant des fonctions entières."

(9) WIMAN (Upsala): "Die metacyklischen Gleichungen neunten Grades."

(10) LOEWY (Freiburg): "Ueber Gruppen linearer homogener Substitutionen." †

(11) STEPHANOS (Athens): "Sur une catégorie d'équations fonctionnelles." †

---

† For the convenience of the reader, a dagger is placed after the titles of the communications for which abstracts are given below.

(12) WILSON (New Haven): "On products in additive fields." †

(13) MÜLLER, E. (Constance): "Ueber den Nachlass von E. Schröder." †

1. Gordan established the normal form of the equation of the sixth degree and gave the differential equation which serves to effect its solution.

3. In Fermat's equation $a^{b-1} = 1 + \alpha b$, where $a$ and $b$ are primes, the integer $\alpha$ may be denoted by $E(a^{b-1}/b)$, if $E(x)$ denote the integral part of the number $x$. Capelli gives the expression

$$\alpha = a\,[1 + E(a^{b-2}/b) + E(b^{a-2}/a)] - b^{a-2},$$

and a similar expression holds for the generalized equation, $a^{\phi(b)} = 1 + \alpha b$, of Fermat, where $a$ and $b$ are merely relative primes. The proof depends on the relation

$$a\,[a^{\phi(b)-1}]_b + b\,[b^{\phi(a)-1}]_a = 1 + ab$$

where $[x]_a$ denotes the positive residue of an integer $x$ taken modulo $a$.

4. The linear factors of an arbitrary form, and hence in particular, of one which contains irreducible factors of higher order are determined by means of the theorem: If there be known a special set of values $x_i = a_i$ $(i = 1, 2, \cdots, n)$ for which a linear factor of a form $f(x_1, x_2, \cdots, x_n)$ vanishes and all the remaining factors are different from zero, then the linear factor is determined by $\Sigma_{i=1}^{i=n} x_i\,(\partial f/\partial x_i)_0$, where the index 0 denotes that the variables $x$ are replaced by the values $a$. If then there be substituted in the equation $f = 0$ the values $x_2 = a_2$, $x_3 = a_3$, $\cdots$, $x_n = a_n$, which are arbitrary save for satisfying the sole condition that the corresponding values $x_1 = a_1^{(1)}$, $x_1 = a^{(2)}$, $\cdots$, $x_1 = a^{(m)}$ are distinct, each linear factor of $f$ is identical with one of the expressions $\Sigma_{i=1}^{i=n} x_i\,(\partial f/\partial x_i)_\lambda$, where the index $\lambda$ denotes that the values $x_1, x_2, \cdots, x_n$ have been replaced by $a_1^{(\lambda)}, a_2, \cdots, a_n$. It must be stipulated that the form $f$ has no multiple factors, or if it has, that these have been removed by Euclid's algorithm. A modification of the method indicated in this theorem for determining the linear factors is based on the theorem: The greatest common divisor of a form and all

three-rowed minors of its hessian determinant is the product of the linear factors of the form. If this product has been found, one has merely to resolve it into its linear factors.

5. Guldberg treats the linear homogeneous difference equation

$$y_{x+n} + p_x^{(1)} y_{x+n-1} + p_x^{(2)} y_{x+n-2} + \cdots + p_x^{(n)} y_x = 0,$$

where the $p_x$'s denote given functions of $x$. He shows that almost all formal theories of linear homogeneous differential equations have their analogues in the theory of such difference equations — for example, the theory of the invariant function, of the transformed equation, of the associated equation, and in particular the fact that the integration of the equation is dominated by the linear homogeneous group of transformations which belongs to the equation.

10. After a short account of his researches on reducibility (published in the *Transactions*), Loewy introduces with Burnside the conception of the complete reducibility of a group of linear homogeneous substitutions. Every such group which transforms a definite Hermite form into itself, is completely reducible. Although the complete reducibility of any *finite* group of linear homogeneous substitutions follows from the theorem, not all *infinite* groups of this type are completely reducible. But by them is uniquely determined in order a series of completely reducible groups in which the total number of variables is equal to the number of variables of the original group. These investigations stand in the closest relation to Frobenius's definition of reducibility of linear homogeneous differential equations, if one introduces the conception of the reducibility of such differential equations as fundamental. This conception is particularly fundamental for considering the totality of irreducible linear homogeneous differential equations whose integrals satisfy a given equation of this type.

11. Stephanos begins with the following fundamental proposition : The necessary and sufficient condition that a function $F(x, y)$ of two variables be expressible by the formula

$$F(x, y) = \Sigma \rho_i(x) x_i(y) \qquad (i = 1, 2, \cdots, m)$$

is the identical vanishing of the determinant

$$\begin{vmatrix} F & \frac{\partial F}{\partial x} & \cdots & \frac{\partial^m F}{\partial x^m} \\ \frac{\partial F}{\partial y} & \frac{\partial^2 F}{\partial x \partial y} & \cdots & \frac{\partial^{m+1} F}{\partial x^m \partial y} \\ \vdots & & & \\ \frac{\partial^m F}{\partial y^m} & \frac{\partial^{m+1} F}{\partial x \partial y^m} & \cdots & \frac{\partial^{2m} F}{\partial x^m \partial y^m} \end{vmatrix}.$$

He applies this proposition to discuss the following questions : 1° what functions $f(x)$ give rise to an addition formula $f(x+y) = \Sigma\phi_i(x)f_i(y)$, 2° what functions give rise to a formula $[f(x) - f(y)]/(x-y) = \Sigma\phi_i(x)\psi_i(y)$, 3° what functions yield $[f(x) - f(y)]/(x-y) = \Sigma\phi_i(x)f_i(y)(x-y)^{i-1}$, 4° when does the development of $[f(x) - f(y)]/(x-y)$ by Lagrange's formula lead to a specified result? The full exposition of these matters will be found in the *Rendiconti di Palermo,* volume 18 (1904).

12. Wilson undertook to give some idea of the views of J. Willard Gibbs on multiple algebra and its connection with vector analysis. It was shown that neither the views and methods of Grassmann nor those of Benjamin Peirce were in themselves sufficiently broad to cover the field. Adopting the more general point of view of Gibbs would, in large measure, allay the present strife between different schools of vector analysis. Above all, it is wrong to infer that the vector analysis represents in any adequate way the totality of Gibbs's work on multiple algebra.

13. Müller reported that in continuation of the Algebra der Logik, of which Schröder had published a first volume and the first parts of a second and third volume before his death, there is shortly to appear the second part of volume 2. The theory of relations, as announced in the volumes which have already appeared, has been left to the second part of the third volume. When this may appear is not yet determined, but first will be published a short account of the algebra of logic as set forth in the first two volumes. This will also appear in the near future.

## Section II. *Analysis.*

Presidents: Schwarz and Hilbert. Chairmen: Mittag-Leffler, Lindelöf, Hadamard, and Levi-Civita.

(1) SCHLESINGER (Klausenburg): "Ueber das Riemannsche Problem zur Theorie der linearen Differentialgleichungen und daran anschliessende neuere Arbeiten."

(2) BOREL (Paris): "Sur l'approximation des fonctions continues par des polynômes." †

(3) HILBERT (Göttingen): "Ueber Integralgleichungen."

(4) VORONOÏ (Warsaw): "Sur le développement, à l'aide des fonctions cylindriques, des sommes doubles $\Sigma f(pm^2 + 2qmn + rn^2)$, où $pm^2 + 2qmn + rn^2$ est une forme quadratique positive à coefficients entiers."

(5) FRICKE (Braunschweig): "Ueber die Existenz der polymorphen Funktionen auf Riemannschen Flächen." †

(6) BOUTROUX (Paris): "Sur les fonctions entières d'ordre entier." †

(7) MITTAG-LEFFLER (Stockholm): "Sur une classe de fonctions entières."

(8) HADAMARD (Paris): "Sur les équations linéaires aux dérivées partielles." †

(9) CAPELLI (Naples): "Ueber die Additionsformeln der Thetafunktionen." †

2. In this communication Borel takes up his researches on the interpolation of continuous functions by means of polynomials. After recognizing that in certain cases Lagrange's formula for interpolation gives a divergent result, he shows how a formula of interpolation convergent for all continuous functions may be obtained. This result has been developed by the author in his Leçons sur les fonctions de variables réelles, Paris, Gauthier-Villars, November, 1904.

5. Given a closed Riemann surface over the plane of the complex variable; upon it consider a function $f(z)$ which, except in $n$ points $z = e_1, e_2, \cdots, e_n$, has the character of a rational function, and undergoes a linear transformation on passing around these points and on describing the periods. Such a function is called linear-polymorphic or briefly polymorphic. In particular are treated such polymorphic functions as map the surfaces (when suitably cut) upon polygons whose sides are fundamental circles, as in the theory of automorphic functions. The theorems of existence given about twenty years ago by Klein and Poincaré were proved by means of considerations of continuity. Yet in many ways these considerations appear incomplete. Fricke spoke on his researches carried out last

year for the purpose of giving a new foundation and development to those considerations of continuity.

6. It is known that the *order* of an infinite product $G(z)$ having the zeros $a_1, a_2, \cdots$ is the smallest number $\rho$ such that the series $\Sigma a_i^{-\rho-\epsilon}$ and $\Sigma a_i^{-\rho+\epsilon}$ shall be respectively convergent and divergent for all values of $\epsilon$. Boutroux seeks to show that, in case $\rho$ is an integer (the case that is left aside as exceptional in the general theory of integral functions), the behavior and the properties of $G(z)$ are similar to those of the exponential function. For example, let $|a_i|$ be proportional to $(i \log i)^{1/p}$. The modulus of $G(z)$ is in general comparable to $e^{Az^p}$, $A$ being a positive function of $|z|$. If there be added to $G(z)$ an integral function $g(z)$ of lower order, the zeros of $G + g$ converge toward $p$ straight lines issuing from the origin and spaced at equal angles.

8. The object of this communication by Hadamard is to show that in the case of equations with real characteristics (the hyperbolic case), as in the case of imaginary characteristics, the method of integration should take as its base the *fundamental solution* (a solution singular at a point and on the characteristic conoid issuing from that point, as is $1/r$ for the equation of Laplace). This solution is the only one which is related in any essential way to the question proposed. The reason that those who have occupied themselves with this question have not been able to generalize in a perfectly satisfactory manner the method of Volterra, is because they started with other solutions, solutions containing parasitic singularities. As to the difficulties which at first sight would seem to nullify the method (the introduction of meaningless integrals), they are merely apparent and may be entirely done away with by properly defining the *finite* part of an infinite integral. The communication ends with a property of Bessel functions and hypergeometric functions — a property which it would be interesting to generalize.

9. In treating the elliptic functions it is very useful to have the various particular formulas for the addition of the $\vartheta$-functions of a single variable combined into one formula with arbitrary characteristics. Hermite in 1858 found such a formula having four terms and restricted to integral characteristics, Betti in 1860 and H. Smith in 1866 gave similar expressions.

Capelli now has reduced this general expression to two terms. If

$$\vartheta_{\gamma, g}(u) = \sum_{n=-\infty}^{n=+\infty} e^{\pi i \omega \left(n+\frac{\gamma}{2}\right)^2 + 2\pi i \left(n+\frac{\gamma}{2}\right)\left(u+\frac{g}{2}\right)}$$

then

$$\vartheta_{\gamma_1, g_1}(u+v) \cdot \vartheta_{\gamma_2, g_2}(u-v) \cdot \vartheta_{\sigma, s}(0) \cdot \vartheta_{\rho, r}(0)$$

$$= \vartheta_{\gamma_1, g_1}(v) \cdot \vartheta_{\gamma_2, g_2}(-v) \cdot \vartheta_{\sigma, s}(u) \cdot \vartheta_{\rho, r}(u) + (-1)^{\sigma} \vartheta_{\gamma_1+1-\sigma, g_1+1-s}(v) \cdot$$

$$\vartheta_{\gamma_2+1-\sigma, g_2+1-s}(-v) \cdot \vartheta_{11}(u) \cdot \vartheta_{\gamma_1+\gamma_2-1, g_1+g_2-1}(u),$$

where $\rho = \gamma_1 + \gamma_2 - \sigma$ and $r = g_1 + g_2 - s$. This formula holds for all real and imaginary values of $\gamma_1, \gamma_2, g_1, g_2$. Only the characteristics $\sigma$ and $s$ need be integers.

## Section III. *Geometry.*

Presidents: Von Brill, Fr. Meyer, and Schur. Chairmen: Zeuthen, Segre, Morley, and Guichard.

(1) Von Brill (Tübingen): "Ueber Elimination und Geometrie seit 50 Jahren." †

(2) Macaulay (London): "Intersection of plane curves." †

(3) Guichard (Clermont-Ferrand): "Sur les systèmes triples orthogonaux."

(4) Study (Bonn): "Kürzeste Wege im komplexen Gebiete." †

(5) Meyer (Königsberg): "Grundzüge einer Theorie des Tetraeders."

(6) Rohn (Dresden): "Ueber algebraische Raumkurven." †

(7) Scheffers (Darmstadt): "Ueber Isogonalkurven, Aequitangentialkurven und komplexe Zahlen." †

(8) Schoenflies (Königsberg): "Struktur der perfekten Mengen." †

(9) Zindler (Innsbruck): "Zur Differentialgeometrie der Liniengeometrie." †

(10) Wilczynski (Berkeley): "The general projective theory of space curves and ruled surfaces." †

(11) Andrade (Besançon): "Les mouvements des solides aux trajectoires sphériques."

(12) Knoblauch (Berlin): "Grundformeln der Theorie der Strahlensysteme." †

(13) Lilienthal (Münster): "Ueber äquidistante Kurven." †

(14) AUTONNE (Lyons): "Sur les substitutions crémoniennes dans l'espace à plusieurs dimensions." †

(15) GENESE (Aberystwith): "On some useful theorems in continued multiplication of the regressive product in the real four point space." †

(16) STUDY (Bonn): "Ueber das Prinzip der Erhaltung der Zahl." †

(20) CABREIRA (Lisbon): "Note sur les rapports polygonaux" (by title).

(21) TIKHOMANDRITZKY (Suida): "Die Winkelsumme eines ebenen Dreiecks" (by title).

(22) GIAMBELLI (Turin): "Sul principio della conservazione del numero" (by title).

1. Numerous geometric problems deal with restricted sets of equations for which all the determinants of a matrix vanish; others lead to the condition expressing the fact that a system of equations which are identically satisfied by one or more sets of values of the variables have also in common another solution. Everything that occurs in elimination is capable of treatment by the aid of a method invented by Kronecker and yielding as many solutions of all kinds as are common to a system of equations. By this method also are to be stated the cases in which all the boundaries within which the principle of conservation of number can be applied without danger — and of this Brill's report gave some examples.

2. The equations which must be identically satisfied by the coefficients of a non-homogeneous polynomial $M$ in $n$ variables, in order that $M$ may be divisible by the modular system $M_1$, $M_2, \cdots, M_n$, are called the linear equations of the manifold $(M)$, which is the intersection of the $n$ manifolds $M_1 = M_2 = \cdots = M_n = 0$. Every proper manifold forming a part of $(M)$ has also its corresponding linear equations. Any such proper manifold which is an isolated point of any degree of multiplicity is called a base point. The paper summarizes the known properties of the linear equations of the intersections of plane curves, and makes some extensions to space of $n$ dimensions.

4. This paper of Study's is concerned with an extension to the imaginary domain of the notion of shortest distance. A method of measurement was developed in which Cayley's

quadratic form is replaced by a Hermite form. The "distance" of two imaginary points in the so-called elliptic case, where the Hermite form is definite, is a real continuum, and can therefore be connected with the notions of "larger" and "smaller." Geodesics can be defined. They lie in straight lines and are identical with a special kind of von Staudt chains. If the coefficients of the Hermite form are real, the geometry based on these notions coincides with elliptic geometry, while in the imaginary domain it has other properties. If $n$ is the number of homogeneous variables, the geometry belongs to a certain group of "motions" and of "transformations of symmetry," the continuous subgroup of which depends upon $n^2 - 1$ parameters and is primitive and simple. The whole of this geometry can be interpreted on a certain manifold of $2n - 2$ dimensions, which is situated in a euclidean space of $n^2 - 1$ dimensions. The imaginary points of a straight line are represented by real points of spheres, all of which have the same radius. Applications were made to the definition of fundamental regions of automorphic functions of $n - 1$ variables, and to the representation of all finite projective groups by means of euclidean motions. An abstract of this rather extensive theory will appear in the *Mathematische Annalen*.

6. The problem of determining all kinds of curves $R_n^p$, of degree $n$ and deficiency $p$ in space has been treated by Noether, Halphen, Valentiner ; but we are still far from a complete insight into the relations which arise. Rohn finds that it is well to study the sets of points in which planes and surfaces of the second, third, up to the $i$th order cut the curve. If these sets are not special, the solution of the problem follows immediately. The solutions may also be brought into a definite form, if the sets are complete special sets. But if they be partial sets, essential difficulties arise and the residual curve must be studied. To every such partial set on $R_n^p$ corresponds an analogous partial set on the residual curve and this is in general no longer special. It may therefore be determined and by it the corresponding set on the curve $R_n^p$.

7. Scheffers sets over against the conformal group in the plane another group consisting of the contact transformations which carry straight lines into straight lines and segments into segments of equal length. This group plays the same rôle for equitangential curves that the conformal group does for isogonal

curves. As the conformal group is given by the equation $x' + iy' = f(x + iy)$, so this new group can be represented by the equation $u' + jv' = f(u + jv)$. But the system of complex numbers founded on the units $(1, j)$ is not the ordinary system for which $i^2 = -1$, but the other system in which $j^2 = 0$.

8. Schoenflies made a report on his memoir in the *Mathematische Annalen*, volume 59.

9. Zindler defines geometrically the direction of advance in a line space and determines it by coördinates. The rays neighboring to a fixed ray $s$ can be arranged into $\infty^3$ directions of line space. Of these only $\infty^2$ are contained in a given complex to which the ray $s$ belongs. They may be represented by $\infty^2$ suitably chosen neighboring rays of the complex, so that there is obtained a visual representation of the division of the rays of a complex in the neighborhood of $s$.

10. Wilczynski gave a brief outline of his investigations during the last few years. These investigations belong to a domain which hitherto has been practically untouched. Projective geometry has, on the whole, confined itself to the consideration of algebraic configurations. The general theory of surfaces, congruences, etc., on the other hand, has not been projective, but metrical. The theory of ruled surfaces and of curves in space as developed by Wilczynski is characterized by the fact that it is a projective theory, applicable not only to algebraic cases but in general. It represents the first chapters of a general projective geometry. The work has all appeared or is shortly to appear in the *Transactions*.

12. The formulas by means of which Bianchi proved Guichard's theorems on the deformation of quadric surfaces and which he has taken up in the new edition of his Geometria differenziale depend upon a particular assumption concerning the parameters of a system of rays and contain two sorts of quantities — those which may be made independent of the choice of the parameters and those which cannot. These disadvantages may be alleviated by making fundamental, not differentiation with respect to the parameters, but those operations which are connected with the theory of a pair of quadratic differential forms and which possess the property of invariance for the system of rays or for its defining surface (Ausgangsfläche). Knoblauch gave an example of the difference between the for-

mulas that arise in this way and those obtained by Bianchi. He reserved the development of the formulas themselves for another place.

13. On pseudospherical surfaces, and only on them, do the asymptotic lines form an equidistant system. In a corresponding manner one may ask if there are surfaces of positive curvature on which the characteristic curves (Encyklopädie, volume 3, part 3, page 115) form an equidistant system. The surfaces in question must be surfaces of translation. In the integration of the five differential equations which arise one must distinguish between Weingarten and non-Weingarten surfaces. For the latter the theorem of existence follows from the actual integration and it is seen that the surfaces in question are isothermal.

14. Autonne regards $2N$ variables $x_i$ and $u_i$ $(i = 1, 2, \cdots, N)$ connected by three relations $\Sigma\, xu = 0$, $\Sigma\, cx = \Sigma\, gu = 1$, ($c_i$, $g_i$ arbitrary numerical constants), as the coördinates of an element $(x, u)$ in a space of $N - 1$ dimensions. The element $(x, u)$ is made up of the point $x$ and the plane $u$, which have the $x_i$'s and the $u_i$'s as their respective homogeneous coördinates. The plane $u$ passes through the point $x$. Let the expression $f(x^m;\, u^{m'})$ denote a polynomial homogeneous in the $x$'s and in the $u$'s, the degrees of homogeneity being respectively $m$ and $m'$. The transformation

$$G = \begin{vmatrix} x_i & \phi_i(x^m;\; u^{m'}) \\ u_i & \psi_i(x^n;\; u^{n'}) \end{vmatrix}$$

will be of Cremona type, if it is 1° birational with respect to the coördinates of an element and 2° a transformation of contact. The general properties of Cremona transformations and the basis of their classification is studied.

15. If $BC$ be a regressive product and if $A$ be chosen so that $AC$ is a scalar, then 1) if $A$ be contained in $B$, $A.\,BC = AC.\,B$; 2) if $A$ be not contained in $B$, $A.\,BC = AC.\,B - AB.\,C$. Theorem 1 was first derived from one of Grassmann's, and then proved geometrically for real space, and its uses shown. Theorem 2 was observed to be true also when $BC$ is not regressive in all cases except one, namely when $A$, $B$, $C$ each represents a line.

16. In this communication Study makes some criticisms on the so-called principle of conservation of number, as stated by Schubert in his "Kalkul der abzählenden Geometrie." The paper will appear in the *Archiv für Mathematik*.

E. B. WILSON.

(*To be continued.*)

---

## NOTES.

THE Annual Register of the AMERICAN MATHEMATICAL SOCIETY for the year 1905 is now in press and will soon be issued. The Register contains this year a complete catalogue of the Society's library, which now amounts to over 2000 volumes. The membership of the Society is now 473, including 32 life-members. The treasurer's report shows a balance of \$ 3884.28 on hand December 27, 1904.

THE presidential address "On mathematical progress in America," delivered by Professor THOMAS S. FISKE at the annual meeting of the Society will be published in the February BULLETIN.

THE annual meeting of the Association of teachers of mathematics in the Middle States and Maryland was held at Princeton, New Jersey, November 26. It was attended by about 100 teachers representing nearly every college and secondary school within its territory. Sixteen new members were admitted, thereby increasing the membership to 297.

The following officers were elected: President, Professor T. S. FISKE; Vice-President, Dean H. B. FINE; Secretary, Dr. ARTHUR SCHULTZE. President Wilson of Princeton University, delivered the address of welcome. The following papers were read: "On defining the various forms of numbers ordinally," by H. B. FINE; "The relative importance of the topics in elementary algebra," by A. HEIKES; "Some of the conditions under which mathematical teachers are working," by M. CURTIS; "The great movements now taking place in the teaching of mathematics in other countries," by G. LEGRAS.

THE Central association of science and mathematics teachers held its fourth meeting at the Northwestern University Profes-