THE FEBRUARY MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

THE one hundred and twenty-second regular meeting of the AMERICAN MATHEMATICAL SOCIETY was held in New York City on Saturday, February 25, 1905. The attendance at the two sessions was about fifty, including the following forty-two members of the Society :

Dr. Grace Andrews, Professor C. L. Bouton, Professor Joseph Bowden, Professor E. W. Brown, Professor F. N. Cole, Professor E. S. Crawley, Miss L. D. Cummings, Dr. D. R. Curtiss, Dr. W. S. Dennett, Dr. L. P. Eisenhart, Dr. William Findlay, Professor T. S. Fiske, Dr. A. S. Gale, Miss Ida Griffiths, Professor E. R. Hedrick, Mr. E. A. Hook, Mr. S. A. Joffe, Dr. Edward Kasner, Dr. O. D. Kellogg, Professor C. J. Keyser, Mr. E. H. Koch, Professor E. O. Lovett, Dr. Emory McClintock, Dr. Max Mason, Professor James Pierpont, Miss Virginia Ragsdale, Miss Amy Rayson, Dr. F. G. Reynolds, Miss I. M. Schottenfels, Professor Charlotte A. Scott, Dr. A. W. Smith, Professor D. E. Smith, Dr. C. E. Stromquist, Professor Henry Taber, Professor J. H. Tanner, Miss M. E. Trueblood, Professor E. B. Van Vleck, Professor J. M. Van Vleck, Professor L. A. Wait, Mr. H. E. Webb, Dr. E. B. Wilson, Dr. Ruth G. Wood.

The Vice-Presidents, Professors Pierpont and E. W. Brown, presided at the morning and afternoon sessions respectively. The Council announced the election of the following persons to membership in the Society : Miss A. F. Becker, Yeatman High School, St. Louis, Mo.; Professor C. H. Beckett, Purdue University ; Professor W. De W. Cairns, Oberlin College ; Professor S. C. Davisson, Indiana University ; Dr. J. S. French, Jacob Tome Institute ; Mr. F. H. Hodge, Clark University ; Mr. A. E. Joslyn, Armour Institute of Technology ; Dr. J. W. Lowber, Austin, Texas ; Mr. J. H. Maclagan-Wedderburn, University of Chicago ; Mr. G. A. Plimpton, New York City ; Mr. E. W. Ponzer, University of Illinois ; Mr. H. W. Reddick, University of Illinois ; Miss M. E. Sinclair, University of Nebraska ; Dr. A. W. Smith, Colgate University. Ten applications for membership were received.

Professor T. S. Fiske desiring to withdraw from the Editorial Committee of the *Transactions* at the completion of the present volume, the vacancy thus created was filled by the election of Professor E. B. Van Vleck.

The following papers were read at this meeting:

(1) Dr. L. D. AMES: "The theorem that a closed simple surface is bilateral."

(2) Professor C. L. BOUTON: "Note on isothermal curves and one-parameter groups of conformal transformations in the plane."

(3) Professor E. W. BROWN: "Note on the variation of the arbitrary and given constants in dynamical equations."

(4) Mr. O. E. GLENN: "Determination of the abstract groups of order p^2qr."

(5) Mr. F. R. SHARPE: "The stability of the motion of a viscous liquid."

(6) Professor JAMES PIERPONT: "Note on infinite products."

(7) Professor CHARLOTTE A. SCOTT: "The elementary treatment of conics by means of the regulus."

(8) Dr. A. W. SMITH: "The symbolic treatment of differential geometry."

(9) Mr. A. M. HILTEBEITEL: "Note on a problem in mechanics."

(10) Mr. R. B. ALLEN: "Hypercomplex number systems with respect to a domain of rationality."

(11) Dr. L. P. EISENHART: "Note on the deformation of surfaces of translation."

Mr. Sharpe's paper was communicated to the Society through Professor E. W. Brown, Mr. Hiltebeitel's through Professor Lovett, and Mr. Allen's through Professor Taber. Mr. Glenn was introduced by Professor Crawley. In the absence of the authors, Dr. Ames's paper was read by Professor Hedrick, Mr. Sharpe's by Professor Brown, Mr. Allen's by Professor Taber, and Mr. Hiltebeitel's paper was read by title. Professor Bouton's paper appears in the present number of the BULLETIN. Abstracts of the other papers follow below. The abstracts are numbered to correspond to the papers in the list above.

1. Darboux (Théorie générale des surfaces, volume 1, page 361) has stated the theorem that a closed unilateral surface has a multiple point, and has suggested a proof for the case in

which the surface can be represented by an algebraic equation of the form $\phi(x, y, z) = 0$. Without assuming that it can be so represented, Dr. Ames proves in this paper that a unilateral smooth closed simple surface cannot exist. If such a surface exists, it can be cut apart and pairs of coincident conical surfaces can be joined to the free edges, so that the total surface is closed and can be treated as a bilateral surface. It is possible to choose a closed curve which cuts the original surface once and only once. By considering the order (cf. BULLETIN, November, 1904, page 581; March, 1904, page 305) of the points of this curve with respect to the enlarged surface, a contradiction is obtained which proves the theorem.

3. The *Transactions* for July, 1903, contains a paper by Professor Brown in which a certain theorem concerning the variation of the known constants in dynamical equations is enunciated. In the present note he shows that the proof of the theorem is faulty, that it is of more limited application and that it requires restrictions in addition to those set down in the previous paper. The theorem in question concerns the possibility of putting $\delta c_i = 0$ in the canonical equations of variations. However, the application to the determination of the secular acceleration of the moon's mean motion is not affected.

4. Mr. Glenn gave an enumeration of all non-isomorphic groups of order p^2qr, and discussed the peculiarities of a number of the types given. Among other results of the investigation the following theorem was enunciated:

A group G_{p^2qr} $(p > q > r)$ always contains a self-conjugate subgroup H of order p^2q. If H is the only maximal invariant subgroup of G, then, except when $r = 2$, H must be abelian. When $r = 2$ there exists one type if $p \equiv 1 \pmod{q}$, and one if $p \equiv -1 \pmod{q}$, whose subgroup H_{p^2q} is not abelian.

An analogous theorem holds for the group G_{p^mqr} whose subgroup I_{p^m} is abelian of type $[1, 1, 1, \cdots (m \text{ units})]$.

5. Professor Reynolds has examined the limit of stability for the steady motion of a viscous liquid flowing symmetrically between two infinite parallel planes. Mr. Sharpe considers this problem by a simpler method and, by assuming a somewhat different type of periodic variation from the steady motion, he obtains a lower limit of stability. He also extends the method

used to a similar problem in the case of a cylindrical tube and deduces a limit of stability which is much lower than that conjectured from general considerations by Professor Reynolds.

6. The uniform and absolute convergence of the infinite product $P = f_1(z)f_2(z) \cdots$ is usually made to depend upon the series $G = g_1(z) + g_2(z) + \cdots$ where $f_n = 1 + g_n\ (n = 1, 2, \cdots)$. Professor Pierpont showed that this question might often be treated more simply without the intervention of the G series.

7. In the usual treatment of conics in projective geometry, the point system of the second order and the line system of the second order are defined independently by means of projective flat pencils and projective ranges; the connection between the two systems is arrived at by somewhat elaborate processes. Miss Scott shows that, by means of the regulus, the point system and line system can be treated together. The polar and involution properties of conics follow immediately, and simple proofs are given for the theorems of Chasles, Brianchon, and Pascal.

8. Dr. Smith's symbolic treatment of differential geometry is based on the investigations of Professor Maschke on differential invariants * and involves notations similar in character and treatment to those employed in the theory of invariants. The quadratic forms of differential geometry are represented symbolically as perfect squares or in the case of congruences as a product of two linear factors. For the practical working of the symbols there is a list of identities more or less independent of the specific meanings of the different symbols. The principal results are these: All invariant forms are visibly such. Equations may be written in a much more compact form, for example, the differential equation of geodesics, which is $(f\phi)(\phi U)(U, (fU)) = 0$. The fundamental magnitudes, for example, the coefficients of the cubic form, are symmetrically expressible symbolically in terms of symbols of lower orders; and the proof of theorems, such as the existence theorem for a surface whose two quadratic forms are given, may be made with perfect generality as to the parameters used, and the desired derivatives actually found.

* "A new method of determining the differential parameters and invariants of quadratic differential quantics," *Trans. Am. Math. Soc.*, 1900. "Invariants of differential quantics," Decennial Publications, University of Chicago, vol. 9.

9. In an article published in the twenty-eighth volume of the *Giornale di Matematiche,* Dr. Carlo Bonacini states with inadequate proof that in the motion of a body under the influence of forces directed towards two fixed centers the variables in the equations of motion can be separated only in case the forces vary inversely as the squares of the respective distances of the body from the fixed centers. The object of Mr. Hiltebeitel's paper is to point out an error in Bonacini's reasoning and to show that the most general law of force depending only on the distance and admitting of the separation of the variables in the equations of motion is a certain linear combination of the laws of direct distance and inverse square of distance first considered by Lagrange in his memoirs on this classic particular case (first studied by Euler) of the problem of three bodies.

10. Mr. Allen's paper is concerned with two problems in the theory of hypercomplex number systems with respect to a domain of rationality:* First, the problem of the determination with respect to a given domain of rationality of all types of hypercomplex number systems in a given number of units; that is, given an arbitrary scalar domain of rationality, to determine all hypercomplex number systems, in a given number n of units into one or other of which any given number system in n units, whose constants of multiplication are included in R, can be transformed by a transformation rational with respect to R. This problem is solved by Mr. Allen for the domain $R \equiv R(1)$ and for $n \geqq 6$. Second, the determination of all number systems $e_1, e_2, \cdots, e_n$ whose constants of multiplication are included in an arbitrary scalar domain R of rationality, and in which division is unambiguous of numbers rational with respect to the hypercomplex domain $\mathbf{R}(R, e_i)$ of rationality, designating as rational with respect to $\mathbf{R}(R, e_i)$ any number $\sum_1^n {}_i a_i e_i$ of the system whose coefficients a_1, a_2, etc., are rational with respect to R. Every such system contains a modulus; and the number of units is determined by the characteristic equation $A^m + p_1(a)A^{m-1} + \cdots + p_{m-1}(a)A + p_m(a) = 0$ of the system. Namely, we have $n = \mu^2\nu$ where $m = \mu\nu$, μ being any factor of m. We may take as units of such a system the products A^pB^q, $(p = 0, 1, 2, \cdots, m;\ q = 0, 1, \cdots,$

* See *Transactions*, vol. 5, p. 511.

$\mu - 1)$ of powers of two properly chosen numbers $A = \sum_1^n {}_i a_i e_i$ and $B = \sum_1^n {}_i b_i e_i$ rational with respect to $\mathfrak{R}(R, e_i)$. Any such system can be transformed into one consisting of ν mutually nilfactorial quadrates of order μ by a transformation rational with respect to the domain obtained by adjoining to R the roots of the equation $x^m + p_1(a)x^{m-1} + \cdots + p_{m-1}(a)x + p_m(a) = 0$ for a properly chosen number $A = \sum_1^n {}_i a_i e_i$ of the system rational with respect to $\mathfrak{R}(Re_i)$.

11. In the January number of the BULLETIN, Dr. Burke Smith proves that the minimal surfaces and surfaces of translation whose generators are in perpendicular planes are the only surfaces of translation which can be deformed in a continuous manner in such a way that the generators continue to be generators. Dr. Eisenhart applies to these surfaces a theorem due to Adam and gets pairs of applicable surfaces of translation with the generators in correspondence; and the equations of these surfaces involve seven arbitrary parameters. The conditions to be satisfied in order that the generators be plane on these new surfaces are given, and a few examples are discussed.

F. N. COLE,
Secretary.

THE DECEMBER MEETING OF THE CHICAGO SECTION.

THE sixteenth regular meeting of the Chicago Section of the AMERICAN MATHEMATICAL SOCIETY was held in the Northwestern University Building, Chicago, on December 30 and 31, 1904. The attendance was thirty-seven, including the following members of the Society:

Mr. R. P. Baker, Professor D. P. Bartlett, Professor G. A. Bliss, Dr. W. H. Bussey, Professor Florian Cajori, Professor D. F. Campbell, Professor L. E. Dickson, Dr. E. L. Dodd, Dr. Saul Epsteen, Professor A. G. Hall, Professor Thomas F. Holgate, Mr. N. J. Lennes, Mr. E. P. Lytle, Professor Heinrich Maschke, Professor G. W. Myers, Mr. Oscar Schmiedel, Miss I. M. Schottenfels, Professor H. E. Slaught, Dr. Burke Smith, Professor E. J. Townsend, Dr. Oswald Veblen, Pro-